Optimization of Gamma Knife Radiosurgery

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Overview

• Details of machine and problem
• Formulation
  - modeling dose
  - shot/target optimization
• Results
  - Two-dimensional data
  - Real patient (three-dimensional) data
The Gamma Knife
What disorders can the Gamma Knife treat?

- Malignant tumors from elsewhere in the body
- Malignant brain tumors
- Benign tumors within the head
- Vascular malformations
- Functional disorders of the brain
  - Parkinson’s disease
201 cobalt gamma ray beam sources are arrayed in a hemisphere and aimed through a collimator to a common focal point.

The patient’s head is positioned within the Gamma Knife so that the tumor is in the focal point of the gamma rays.
How is Gamma Knife Surgery performed?

Step 1: A stereotactic head frame is attached to the head with local anesthesia.
Step 2: The head is imaged using a MRI or CT scanner while the patient wears the stereotactic frame.
Step 3: A treatment plan is developed using the images. **Key point:** very accurate delivery possible.
Step 4: The patient lies on the treatment table of the Gamma Knife while the frame is affixed to the appropriate collimator.
Step 5: The door to the treatment unit opens. The patient is advanced into the shielded treatment vault. The area where all of the beams intersect is treated with a high dose of radiation.
Gamma Knife Statistics

- 120 Gamma Knife units worldwide
- Over 20,000 patients treated annually
- Accuracy of surgery without the cuts
- Same-day treatment
- Expensive instrument
Treatment Planning
Treatment Planning

• Through an iterative approach we determine:
  - the number of shots
  - the shot sizes
  - the shot locations
  - the shot weights

• The quality of the plan is dependent upon the patience and experience of the user
1 Shot
2 Shots
3 Shots
4 Shots
Automated Planning

- Develop a fully automated approach to Gamma Knife treatment planning
- Better tumor dose coverage
- Reduced dose to normal tissue
- More efficient treatments
- Reduced time commitment for neurosurgeon
Computational Model

- Target volume (from MRI or CT)
- Maximum number of shots to use
  - Which size shots to use
  - Where to place shots
  - How long to deliver the shot

- Conform to Target (50% isodose curve)
- Real-time optimization
Ideal Optimization

\[
\begin{align*}
\min_{t_{s,w,x,y}} & \quad Dose(NonTarget) \\
\text{subject to } & \quad Dose(i, j) = \sum_{s \in S, w \in W} t_{s,w}D_w(x_s, y_s, i, j) \\
& \quad 0.5 \leq Dose(Target) \leq 1 \\
& \quad t_{s,w} \geq 0 \\
& \quad |S| \leq N
\end{align*}
\]
Dose calculation

• Measure dose at distance from shot center in 3 different axes
• Fit a nonlinear curve to these measurements (nonlinear least squares)
• Functional form from literature, 10 parameters to fit via least-squares

\[ m_1 \text{erf}\left(\frac{d_1(x)}{\sigma_1} r_1\right) + m_2 \text{erf}\left(\frac{d_2(x)}{\sigma_2} r_2\right) \]
Environment

• All data fitting and optimization models formulated in GAMS
  - Ease of formulation / update
  - Different types of model
• Nonlinear programs solved with CONOPT (generalized reduced gradient)
• LP’s and MIP’s solved with CPLEX
Nonlinear Approach

Let $x_s, y_s$ be variable locations

$s = 1, 2, \ldots, N$

$D_w(x_s, y_s, i, j)$ is nasty nonlinear function

What width shot to use at $x_s, y_s$?

$$\psi_{s,w} = \begin{cases} 1 & \text{if shot s is width w} \\ 0 & \text{else} \end{cases}$$

$$T\psi_{s,w} \leq t_{s,w} \leq \overline{T}\psi_{s,w}$$

$$\sum_w \psi_{s,w} \leq 1$$
Iterative Approach

- Approximate via “arctan”

\[ \forall s \in S \]
\[ \sum_w \arctan(t_{s,w}) \leq \frac{\pi}{2} \]

- First, solve with coarse approximation, then refine and reoptimize
Continuation Approach

- Rotate data (prone/supine)
- Conformity subproblem (P)
- Coarse grid shot optimization
- Refine grid (add violated locations)
- Refine smoothing parameter
- Round and solve MIP for exposure times
Patient 1 - Axial Image
Patient 1 - Coronal Image
manual

optimized

Reconstructed, y: 89.7

Reconstructed, y: 89.7
Patient 2 - Axial slice

15 shot manual

12 shot optimized
Patient 3

optic chiasm

pituitary adenoma
The graph shows the relationship between fraction of volume and dose (Gy) for two different regions: the chiasm and the tumor. The yellow line represents the forward dose response for both regions, indicating the decrease in fraction of volume with increasing dose.
Patient 4 - Axial slice

7 shot manual

7 shot optimized
Skeletons (J.-H. Lim)

a. Target area

b. A single line skeleton of an image

c. 8 initial shots are identified

d. An optimal solution: 8 shots

1-4mm, 2-8mm, 5-14mm
## Starting Point Comparison

<table>
<thead>
<tr>
<th>Average Run Time</th>
<th>Size of Tumor</th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Medium</td>
<td>Large</td>
<td></td>
</tr>
<tr>
<td>Random (Std. Dev)</td>
<td>2 min 33 sec</td>
<td>17 min 20 sec</td>
<td>373 min 2 sec</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(40 sec)</td>
<td>(3 min 48 sec)</td>
<td>(90 min 8 sec)</td>
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</tr>
<tr>
<td>SLSD (Std. Dev)</td>
<td>1 min 2 sec</td>
<td>15 min 57 sec</td>
<td>23 min 54 sec</td>
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</tr>
<tr>
<td></td>
<td>(17 sec)</td>
<td>(3 min 12 sec)</td>
<td>(4 min 54 sec)</td>
<td></td>
</tr>
</tbody>
</table>
Status

• Automated plans have been generated retrospectively for over 30 patients
• The automated planning system is now in use at U. Maryland Hospital
• Head to head against the neurosurgeon
Robustness

- High-quality conformal dose distributions are obtained independent of starting point
- Optimization performs well for targets over a wide range of sizes and shapes
Conclusions

• An automated treatment planning system for Gamma Knife radiosurgery has been developed using optimization techniques (GAMS, CONOPT and CPLEX)
• The system simultaneously optimizes the shot sizes, locations, and weights
• Automated treatment planning should improve the quality and efficiency of radiosurgery treatments