



# GAMS

## Transportation Model

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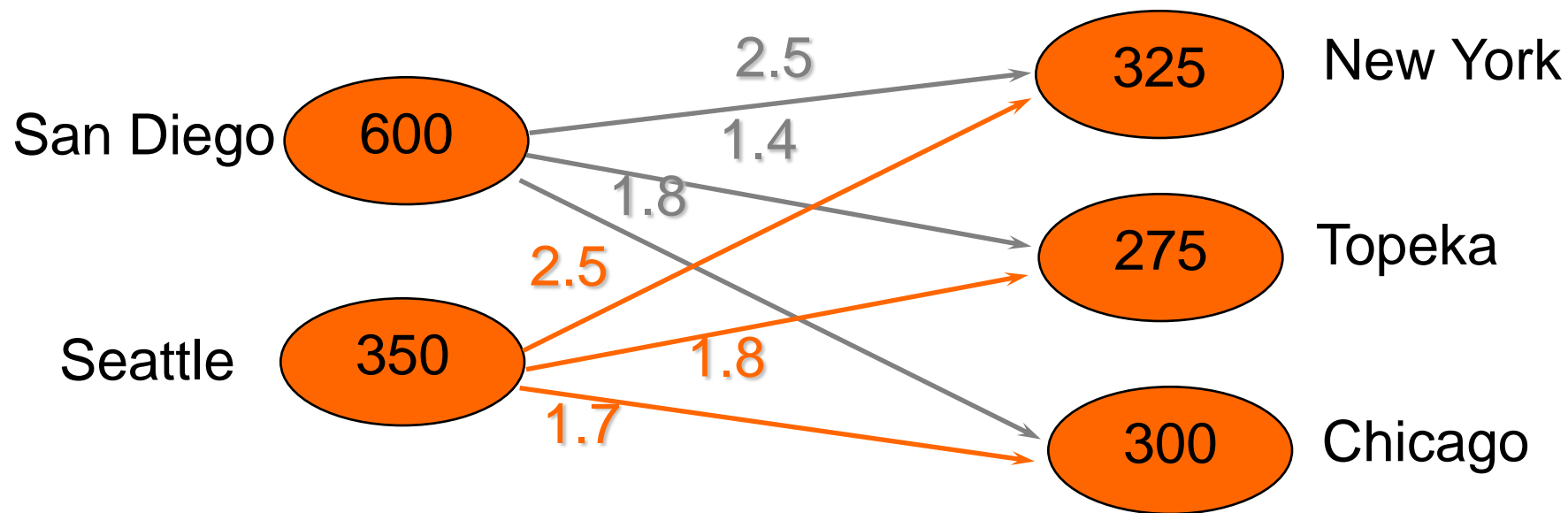


# What is a Model?

- Mathematical Programming (MP) Model
  - List of Equations
- Collection of several intertwined MP Models
  - Data Preparation
  - Data Calibration
  - “Solution” Module (e.g. sequential, parallel, loop)
  - Report Module



# A Transportation Model



**Minimize** Transportation cost  
**subject to** Demand satisfaction at markets  
 Supply constraints



# Mathematical Algebra

$$\sum_{\substack{c,p: \\ (c,p) \in \mathcal{N}}} tcost \cdot dist(c,p) \cdot x_p^c \rightarrow \min$$

$$\sum_{\substack{c,p: \\ (c,p) \in \mathcal{N}}} x_p^c \leq sup(c) \quad \forall c$$

$$\sum_{\substack{c,p: \\ (c,p) \in \mathcal{N}}} x_p^c \geq dem(p) \quad \forall p$$

$$x_p^c \geq 0 \quad \forall c, p : (c, p) \in \mathcal{N}$$



# GAMS Algebra

```
IDE gamside: C:\Documents and Settings\bussieck\My Documents\gamsdir\project.gpr - [c:\documents an...
IDE File Edit Search Windows Utilities Help
call
transport.gms

Variables
    x(i,j)  shipment quantities in cases
    z       total transportation costs in thousands of dollars ;

Positive Variable x ;

Equations
    cost          define objective function
    supply(i)     observe supply limit at plant i
    demand(j)    satisfy demand at market j ;

cost ..          z =e= sum((i,j), c(i,j)*x(i,j)) ;

supply(i) ..     sum(j, x(i,j)) =l= a(i) ;

demand(j) ..     sum(i, x(i,j)) =g= b(j) ;

Model transport /all/ ;
```



# A few Word about GAMS Syntax

- Symbols:

- Sets
- Parameters
- Variables
- Equations
- Models
- ASCII Output Files

```

Sets          i          canning plants / seattle, san-diego /;
Parameters   a(i)       capacity of plant i in cases
                /
                seattle    350
                san-diego  600 /;
Variables    x(i,j)     shipment quantities in cases;
Equations    supply(i)  observe supply limit at plant i;
Model        transport /all/ ;
File         fx          some file / 'c:\t\text.txt' /
    
```

- Statements

- Declarations
- Data Assignments
- Equation Definition
- Programming Flow Control
- Option statement

```

Parameter c(i,j);
c(i,j) = f * d(i,j) / 1000 ;
supply(i) .. sum(j, x(i,j)) =l= a(i);
loop(i, put fx i.t1);
option reslim=10;
    
```





# Hands-on! Transportation Model

```

Parameters
  a(i)  capacity of plant i in cases
        /
        seattle  350
        san-diego 600 /

  b(j)  demand at market j in cases
        /
        new-york  325
        chicago   300
        topeka    275 / ;

Table d(i,j)  distance in thousands of miles
              new-york  chicago  topeka
seattle      2.5        1.7        1.8
san-diego    2.5        1.8        1.4 ;

Scalar f  freight in dollars per case per thousand miles /90/ ;

Parameter c(i,j)  transport cost in thousands of dollars per case
    
```

Seq#	Name	Application Area	Type	Contributor	Description
001	TRANSPORT	Management Science and OR	LP	Dantzig, G.B.	A Transportation Problem
002	BLEND	Management Science and OR	LP	Dantzig, G.B.	Blending Problem I
003	PRODMIX	Management Science and OR	LP	Dantzig, G.B.	A Production Mix Problem
004	WHOUSE	Management Science and OR	LP	Dantzig, G.B.	Simple Warehouse Problem
005	JOBT	Management Science and OR	LP	Dantzig, G.B.	On-the-Job Training
006	SROUTE	Management Science and OR	LP	Dantzig, G.B.	The Shortest Route Problem
007	DIET	Micro Economics	LP	Dantzig, G.B.	Stigler's Nutrition Model
008	AIRCRAFT	Management Science and OR	LP	Dantzig, G.B.	Aircraft Allocation Under Uncertain Demand
009	PRODSCH	Management Science and OR	MIP	CDC	APEX - Production Scheduling Model
010	FDI	Management Science and OR	LP	ARCNET	ARCNET - Production Distribution and Inventory
011	UIMP	Management Science and OR	LP	Elkorn, E.F.	UIMP - Production Scheduling Problem
012	MAGIC	Management Science and OR	MIP	Garver, L.L.	Magic Power Scheduling Problem
013	FERTS	Micro Economics	LP	Chokri, A.M.	Egypt - Static Fertilizer Model
014	FERTD	Micro Economics	MIP	Chokri, A.M.	Egypt - Dynamic Fertilizer Model
015	MEXSS	Micro Economics	LP	Kendrick, D.	Mexico Steel - Small Static
016	MEXSD	Micro Economics	MIP	Kendrick, D.	Mexico Steel - Small Dynamic
017	MEXLS	Micro Economics	LP	Kendrick, D.	Mexico Steel - Large Static
018	WEAPONS	Management Science and OR	NLP	Bracken, J.	Weapons Assignment
019	BID	Micro Economics	MIP	Bracken, J.	Bid Evaluation
020	PROCESS	Chemical Engineering	NLP	Bracken, J.	Alkylation Process Optimization
021	CHEM	Chemical Engineering	NLP	Bracken, J.	Chemical Equilibrium Problem
022	SHIP	Engineering	NLP	Bracken, J.	Structural Optimization
023	LINEAR	Econometrics	DNLP	Bracken, J.	Linear Regression with Various Criteria
024	LEAST	Econometrics	NLP	Bracken, J.	Nonlinear Regression Problem
025	LIKE	Econometrics	NLP	Bracken, J.	Maximum Likelihood Estimation
026	CHANCE	Agricultural Economics	NLP	Bracken, J.	Chance Constrained Feed Mix Problem
027	SAMPLE	Statistics	NLP	Bracken, J.	Stratified Sample Design
028	PINDYCK	Energy Economics	NLP	Pindyck, R.S.	Optimal Pricing and Extraction for OPEC
029	ZLOOF	Management Science and OR	GAMS	Zloof, M.M.	Relational Database Example
030	VIETMAN	Micro Economics	MIP	Manne, A.S.	Vietorissz Manne Fertilizer Model 1961
031	ALUM	International Trade	MIP	Brown, M.	World Aluminum Model
032	MARCO	Micro Economics	LP	Aronofsky, J.	Mini Oil Refining Model

```

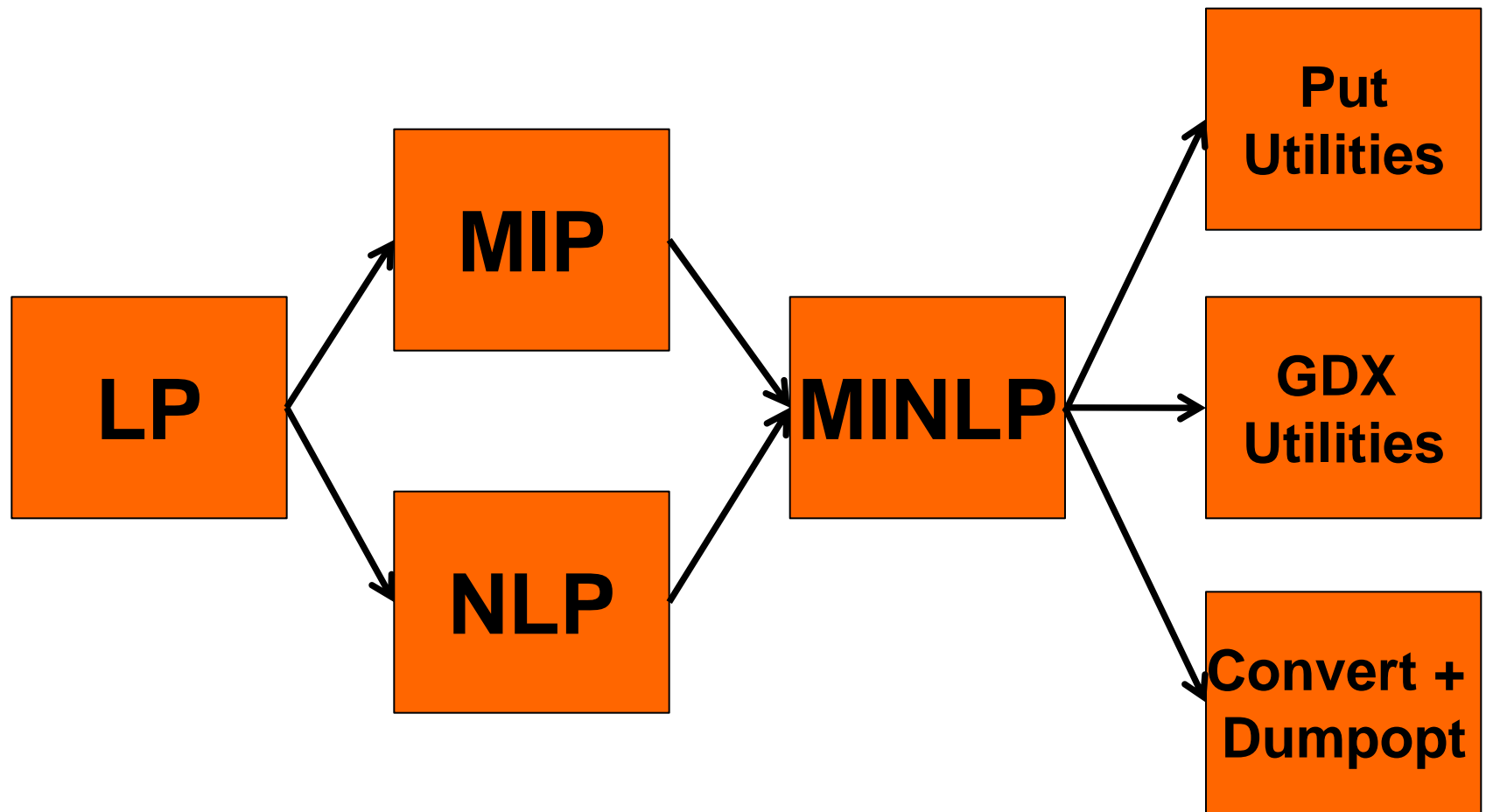
transport

--- Job transport.gms Start 07/03/07 10:25:45
GAMS Rev 148 Copyright (C) 1987-2007 GAMS Development. All rights reserved.
Licensee: Jan-Hendrik Jagla                                G07D418/0001C
              GAMS Software GmbH
--- Starting compilation
--- transport.gms(69) 3 Mb
--- Starting execution
--- transport.gms(45) 4 Mb
--- Generating LP model transport
--- transport.gms(66) 4 Mb
--- 6 rows 7 columns 19 non-zeros
--- Executing CPLEX

GAMS/Cplex Jun 1, 2007 WIN.CP.CP 22.5 034.037.041.VIS For Cplex 10
Cplex 10.2.0, GAMS Link 34
    
```



# Modifications to the transport model







# Types of Variables

- Continuous Variables
  - Free/Positive/Negative
  - Lower and/or upper bound
- Binary Variables
  - Either 0 or 1
- Integer Variables
  - by default ranging between 0 and 100
- Semicont/Semiint Variables
  - 0 or above a given minimum
- Special Ordered Set Variables (SOS1, SOS2)



# Binary Variables

- Powerful Tool to model yes/no decisions
- Models with discrete variables (MIP)
  - Solved using Branch-and-Cut algorithms (lots of LPs)
  - Theoretically difficult problem class
  - Practical:
    - mixed bag
    - *Art of Modeling*
- Example: Minimum Shipment
  - Ship at least 100 tons or don't ship



# Minimum Shipment

- Continuous Variable  $x$  (shipment)
- Binary Variable  $ship$  (indicator for minimum shipment):
  - $ship=1$  if  $x \geq 100$
  - $ship=0$  if  $x = 0$
- Coupling Constraints:
  - $x \geq 100 * ship$
  - $x \leq bigM * ship$
- How big do we have to make bigM?



# Implement Min/Max Shipments (MIP)

```
Parameter rep1(i,j,*)    Shipments between plants and markets
           rep2(*)       Objective value;
```

```
rep1(i,j,'lp') = x.l(i,j);
rep2('lp')     = z.l;
```

```
scalars xmin / 100 /
        xmax / 325 /;
```

```
binary variables ship(i,j)    decision variable to ship
equations      minship(i,j) minimum shipments
              maxship(i,j) maximum shipments ;
```

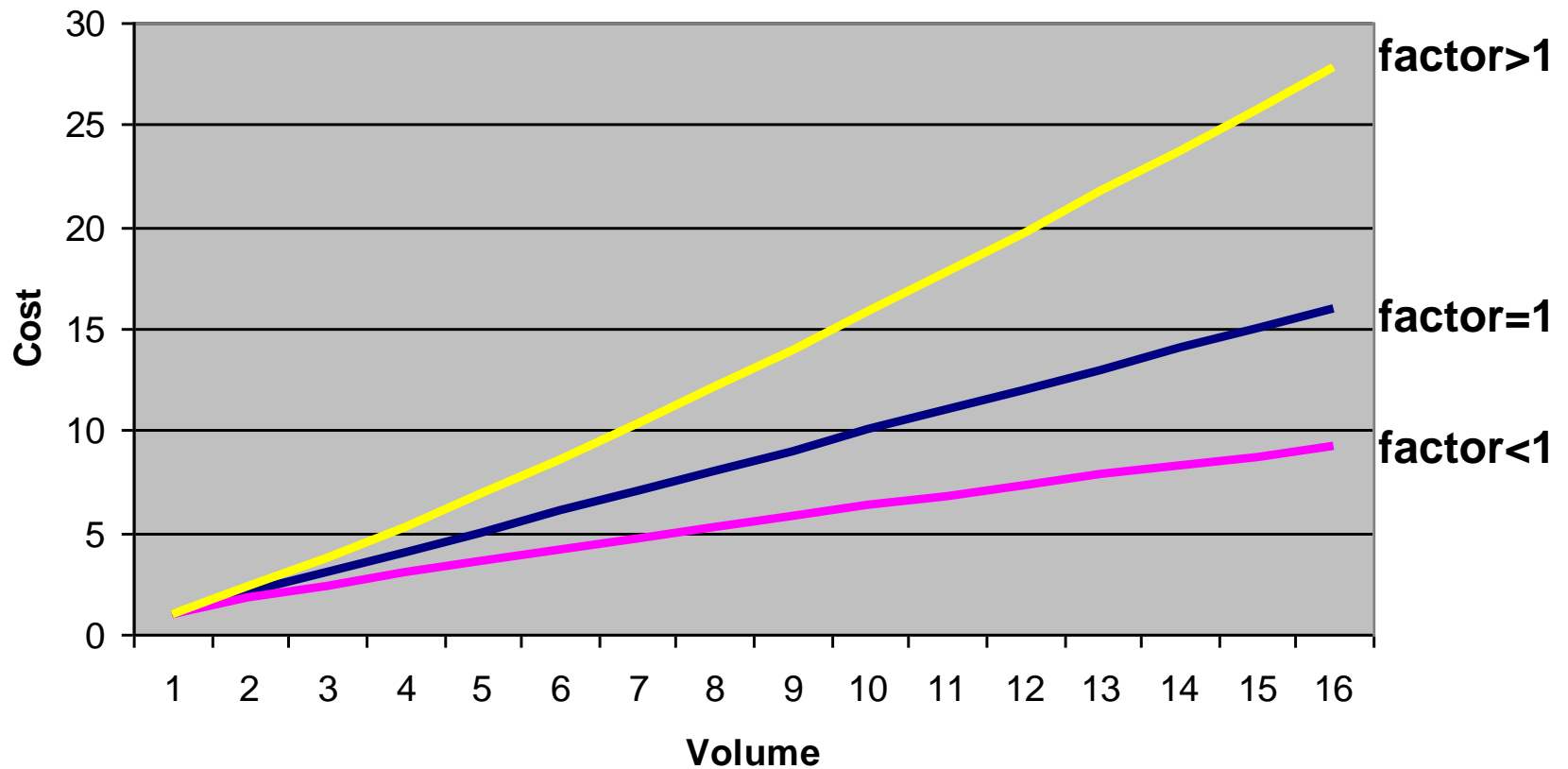
```
minship(i,j).. x(i,j) =g=  xmin*ship(i,j);
maxship(i,j).. x(i,j) =l=  xmax*ship(i,j);
```

```
model m2 min shipments / all /;
solve m2 using mip minimizing z;
rep1(i,j,'mip') = x.l(i,j);
rep2('mip')     = z.l;
```

```
option mip=coincbc
solve m2 using mip minimizing z;
rep1(i,j,'mip-coincbc') = x.l(i,j);
rep2('mip-coincbc')     = z.l;
display rep1,rep2;
```



# Economy of Scales: $\text{Cost} = \text{Volume}^{\text{factor}}$







# Implement Nonlinear Cost (NLP)

```
* nonlinear cost
equation nlcost nonlinear cost function;
scalar beta;

nlcost.. z =e= sum((i,j), c(i,j)*x(i,j)**beta);

model m3 / nlcost,supply,demand /;

beta = 1.5;
solve m3 using nlp minimizing z;
rep1(i,j,'nlp-convex') = x.l(i,j);
rep2('nlp-convex')      = z.l;

beta = 0.6;
solve m3 using nlp minimizing z;
rep1(i,j,'nlp-concave') = x.l(i,j);
rep2('nlp-concave')     = z.l;

option nlp=baron;
solve m3 using nlp minimizing z;
rep1(i,j,'nlp-baron') = x.l(i,j);
rep2('nlp-baron')     = z.l;

display rep1,rep2;
```



# Implement Min/Max and Nonlinear (MINLP)

```
* min/max and nonlinear objective
```

```
model m4 / nlcost,supply,demand,minship,maxship /;
```

```
option minlp=baron;
```

```
solve m4 using minlp minimizing z;
```

```
rep1(i,j,'minlp-bar') = x.l(i,j);
```

```
rep2('minlp-bar')      = z.l;
```

```
option minlp=lindoglobal;
```

```
solve m4 using minlp minimizing z;
```

```
rep1(i,j,'minlp-lin') = x.l(i,j);
```

```
rep2('minlp-lin')      = z.l;
```

```
display rep1,rep2;
```



# Data Connectivity

- Data Import/Export from *Standard Applications*
  - Text files
  - Gams Data eXchange (GDX)
    - MS Office, Databases, ...
- Capture an *Instance*
  - Reproducibility of Model/System Bugs
  - Problems: Life Database/different Platforms
  - convert
  - dumpopt



# Put Utility

```
set help(*)
option help<rep1;

file fx /results.txt/;

put fx 'Results of different models created on ' system.date /;
put '-----' / /;
loop(help,
  put 'Model:' help.te(help) /;
  put '-----' / /;
  put 'Objective value:' rep2(help) / /;
  loop((i,j)$rep1(i,j,help),
    put 'Shipment from 'i.te(i):10' to 'j.te(j):10' is: 'rep1(i,j,help) /;
  );
  put / /;
);
putclose;
```



# GDX and GDXRW

- execute\_unload 'all.gdx';
- .gdx=all2

→ gdxdiff

Entry	Symbol	Type	Dim	Nr Elem
1	rep1	Par	3	31
2	rep2	Par	1	7

rep1								
Plane Index (empty)								
		lp	mip	nlp-convex	nlp-non	nlp-baron	minlp-bar	minlp-lin
seattle	new-york	50	150	142.384077867818			99.9999999998	100
	chicago	300	200	130.929935459408	300	300	200	200
	topeka			76.6859866727732				
san-diego	new-york	275	175	182.615922132182	325	325	225	225
	chicago		100	169.070064540592			100	100
	topeka	275	275	198.314013327227	275	275	275	275

reports.xls - Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K
1			lp	mip	nlp-conve	nlp-non	nlp-baron	minlp-bar	minlp-lin		
2			153.675	154.575	1983.555	15.58477	15.58477	19.27396	19.27396		
3											
4			lp	mip	nlp-conve	nlp-non	nlp-baron	minlp-bar	minlp-lin		
5	seattle	new-york	50	150	142.3841			100	100		
6	seattle	chicago	300	200	130.9299	300	300	200	200		
7	seattle	topeka			76.68599						
8	san-diego	new-york	275	175	182.6159	325	325	225	225		
9	san-diego	chicago		100	169.0701			100	100		
10	san-diego	topeka	275	275	198.314	275	275	275	275		
11											
12											
13											
14											
15											

- execute\_unload 'reports.gdx' rep1, rep2;
- execute 'gdxrw reports.gdx par=rep2 cdim=1 rdim=0 rng=Report!c1';
- execute 'gdxrw reports.gdx par=rep1 cdim=1 rdim=2 rng=Report!a4';





# Capture an Instance with `convert` /`dumpopt`

- GAMS “solver”: `convert`
  - `gams mymodel modeltype=convert`
  - or
  - `option minlp=convert;`  
`solve m4 using minlp minimizing z;`
  - anonymized scalar model `gams.gms` and dictionary `dict.txt`
- GAMS option: `dumpopt`
  - `gams mymodel dumpopt=11`
  - `mymodel.dmp` with all source and data