CGE Models in GAMS: History and Current State of the Art

Sherman Robinson

September 19, 2003

History: Solving CGE Models

- Various approaches
  - Programming models
    - LP: Chenery, Evans, Norton-Scandizzo
    - NLP: Ginsburgh-Waelbroeck, Ginsburgh-Kayzer
  - Systems of equations: “square models”
    - Scarf, Shoven-Whalley
    - Johansen, Dixon (Orani), Hertel (GTAP)
    - Adelman-Robinson, Dervis-deMelo-Robinson

History: Solving CGE Models

- The Scarf algorithm died out quickly.
  - Empirically very inefficient and impractical.
- The programming approaches are still around, but little used.
- Direct solution approaches have prevailed. Basically, two schools:
  - ORANI: CGE model in log differentials.
  - Direct methods: CGE model in “levels”.

Solving CGE Models in Levels

- Tatonnnement methods
  - Gauss-Seidel/Jacobi algorithms
- Jacobian methods
  - Newton-Rapheson
  - M.J.D. Powell: Powell algorithm
- Fortran programs
  - 1970s and early 1980s

CGE Models in GAMS

- Mid 1980s: convert to GAMS
- Modeling “systems”
  - Hercules (Pyatt-Drud)
  - MPSGE (Rutherford)
- “Cameroun” model
  - Devarajan; Robinson (USDA/ERS)
  - Devarajan, Lewis, Robinson: a “standard” CGE model

CGE Models in GAMS

- IFPRI "standard" CGE model
  - Lofgren, Harris, and Robinson
- SAM-based model
  - New UN-SNA SAM
  - CGE model consistent with new data framework

IFPRI CGE Model in GAMS

- The paper is accompanied by a modeling system: a set of GAMS input files (model, data, simulations, reports)
- The system includes extensive reports.
- The system has been applied to a large number of countries.

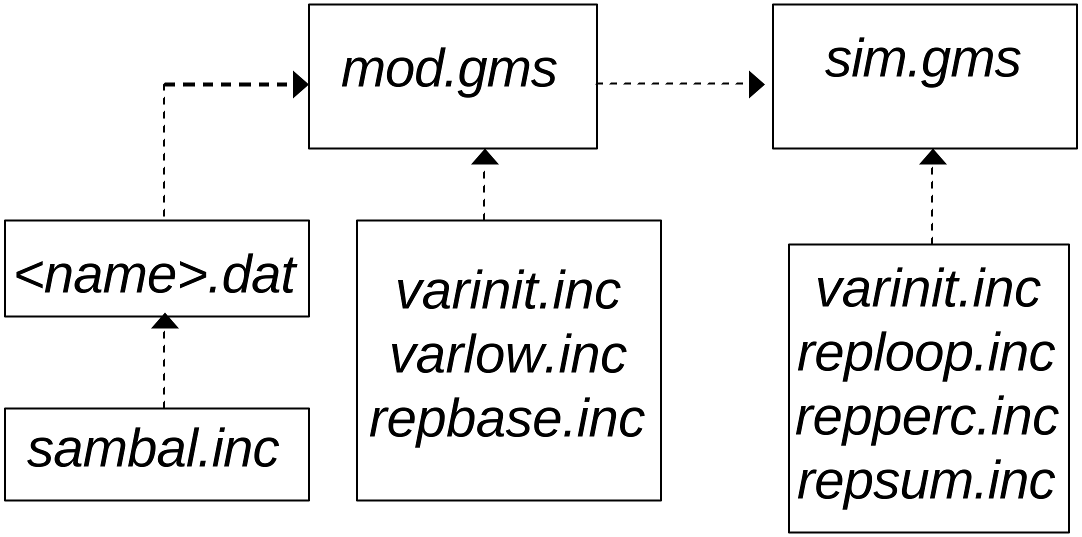
Application Countries

- Argentina
- Bangladesh
- Bolivia
- Botswana
- Brazil
- Chile
- China
- Colombia
- Costa Rica
- Dominican Republic
- Ecuador
- Egypt
- Honduras
- Indonesia
- Jamaica
- Madagascar
- Malawi
- Mexico

Application Countries

- Morocco
- Mozambique
- North Korea
- Paraguay
- Philippines
- Peru
- Russia
- South Africa
- Swaziland
- Tanzania
- Thailand
- Turkey
- Uganda
- United States
- Uruguay,
- Venezuela
- Vietnam
- Zambia
- Zimbabwe

GAMS Modeling System



The Model in GAMS

- This system may be used in the following ways:
  1. Simulations with existing data set without changing the modeling structure (*sim.gms*).
  2. Simulations with new data set:
     a. construct data set (<*name*>*.dat*)
     b. simulations (*sim.gms*)
  3. Simulations with new data set and model structure:
     a. construct data set (<*name*>*.dat*)
     b. adjust model structure (*mod.gms*) and selected include files (*?.inc*)
     c. simulations (*sim.gms*)

GAMS Solvers for CGE Models

- NLP Solvers
  - Minos: Stanford
  - Conopt: Drud
  - Used to solve "square" CGE models or models expressed as NLP problems
- MCP: "Square" solvers
  - Miles: Rutherford
  - Path: Ferris, Munson, Dirkse

Algorithms and Models

- Interaction between developers of CGE models, developers of solution algorithms, and GAMS language features.
- Productive interaction.
  - Special properties of CGE models.
  - Test problems.

CGE Model

```
(1) QS(a) =E=
        aq(a)*PROD(f,QF(f,a)**betaq(f,a));
(2) WF(f)*QF(f,a) =E= betaq(f,a)*PQ(a)*QS(a);
(3) YH            =E= SUM(f, WF(f)*qfs(f)) ;
(4) PQ(a)*QD(a)   =E= betah(a)*YH ;
(5) QD(a) – QS(a) =E= 0 ;
(6) SUM((f,a), QF(f,a)) – qfs(f) =E= 0 ;
(7) SUM(a, alpha(a)*PQ(a))        =E= pbar ;
```

Identities:

```
YH          ≡ SUM(a, PQ(a)*QD(a))
PQ(a)*QS(a) ≡ SUM(f, WF(f)*QF(f,a))

SUM(a, PQ(a)*(QD(a)-QS(a)) +
SUM(f, WF(f)*(SUM(a, QF(f,a))-qfs(f)) ≡ 0
```

Equations and Variables

Equation:	Variables:
(1) Product supply	QS(a)
(2) Factor demand	QF(f,a)
(3) Income	YH
(4) Product demand	QD(a)
(5) Product market	PQ(a)
(6) Factor market	WGF(f)
(7) Price numeraire	

Square CGE Model

- The model has one more equation than number of variables.
- The third identity (Walras' Law) implies that the excess demand equations are functionally dependent.
  - Jacobian is singular.
- Drop one excess demand equation.
  - MCP will solve this model.
  - NLP solvers will also work fine, with arbitrary objective function.

NLP Solvers

- Problem is to find unique feasible basis.
- Objective function is irrelevant, but judicious choice of objective function may improve solver performance.
  - Sum of squared excess demands.
- Feasible to loosen model and optimize.
  - Social welfare function.

Benign Singularity

- Add an identity. One more equation than number of variables.
  - Or do not drop an excess demand equation.
- The solution is unchanged—benign singularity.
- NLP solvers will solve this model.
  - Possible scaling/precision problems, but new solvers seem unfazed.
- MCP solver will fail—model is not square.

Benign Rectangularity

- Some factors do not appear in some sectors. They are excluded from production functions and factor demand equations.
- But, perhaps not excluded from factor market clearing equations.
- Result: model with excess variables.

Benign Rectangularity

```
(1)QS(a) =E= aq(a)*PROD(f$betaq(f,a),
    QF(f,a)**betaq(f,a));

(2) $betaq(f,a)..
    WF(f)*QF(f,a) =E= betaq(f,a)*PQ(a)*QS(a) ;

(6) SUM((f,a), QF(f,a)) – qfs(f) =E= 0 ;
```

Benign Rectangularity

- Excess variables must all be zero at solution. So, solution does not change.
- NLP solvers will solve this model.
- MCP will fail—model is not square.

MCP Test

- Add one identity and one benign excess variable.
  - Benign singularity.
- Model is square, but Jacobian is singular.
- NLP solvers will solve this model.
- MCP will also solve it.

Walras' Law

- Walras' Law implies a singular Jacobian of excess demand equations.
  – Standard approach: drop one equation.
- Alternative: Add a dummy variable
  – "Walras" variable must be zero at solution.
  – Model is square.
- Jacobian is singular at solution.
- NLP and MCP solvers all work.

Conclusion

- MCP solvers
  - Facilitate development of CGE models incorporating complementarity relationships
    - Minimum wage
    - TRQs and import rationing
    - LP technology specification
    - Agricultural policy regimes
    - “Regime shift” macro models

Conclusion

- NLP solvers
  - Dynamic optimization: optimal growth models.
    - Rational expectations models.
    - Macro models.
  - Welfare analysis: welfare maximization with policy instruments.