

NLP Reformulation of MINLP under Nonlinear Network Dynamics

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Outline

- Application Background
- Problem Classes
- Reformulation Techniques
- Conclusions

Application

Projects

Partners

BWB – Berliner Wasserbetriebe (Berlin)

ABB Utilities GmbH (Mannheim)

Bernd Gnädig, M. Steinbach (ZIB)

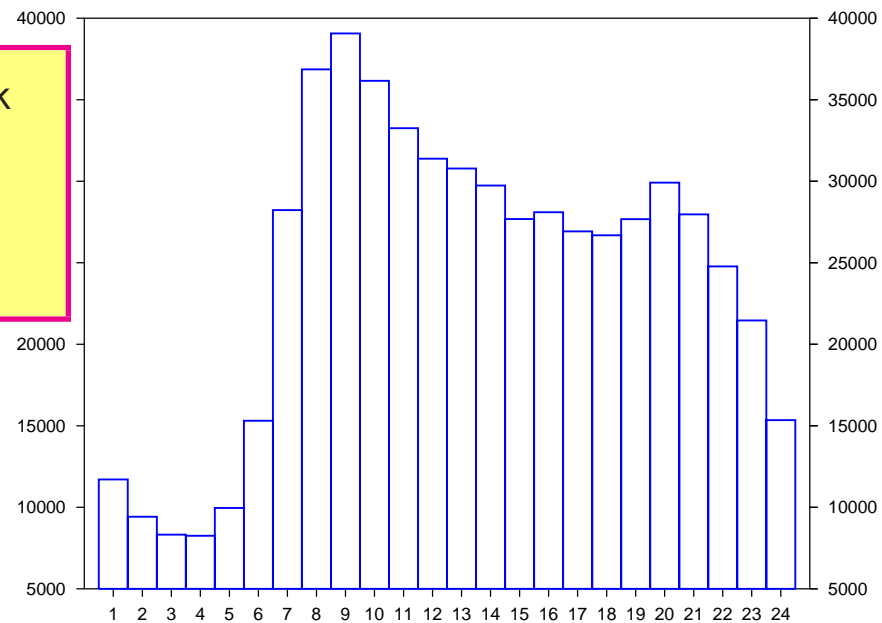
Ruhrgas AG (Essen)

PSI AG (Berlin)

Klaus Ehrhardt, M. Steinbach (ZIB)

Operative Planning

Find: Operating schedule of network satisfying the predicted demand subject to physical, technical, contractual, and other restrictions



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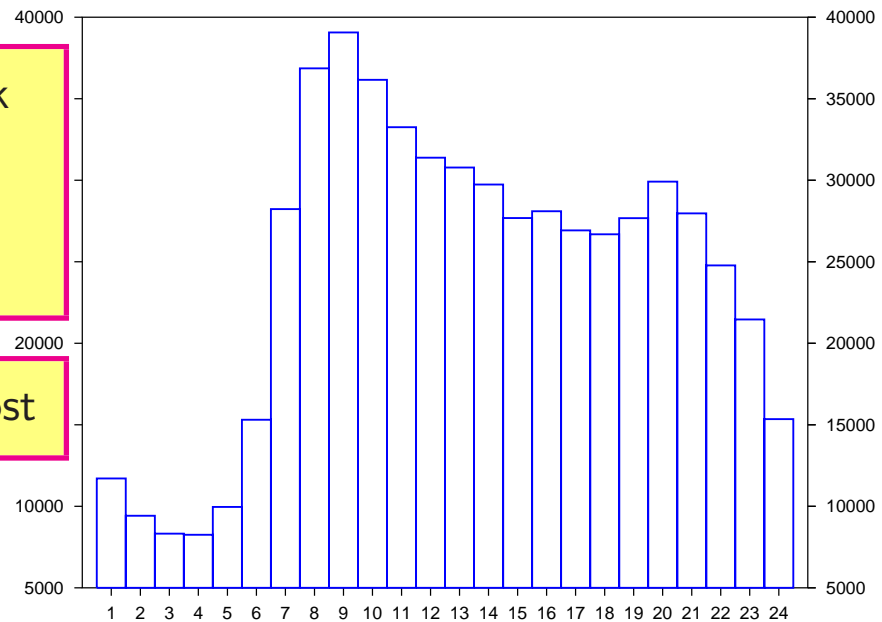
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Operative Planning

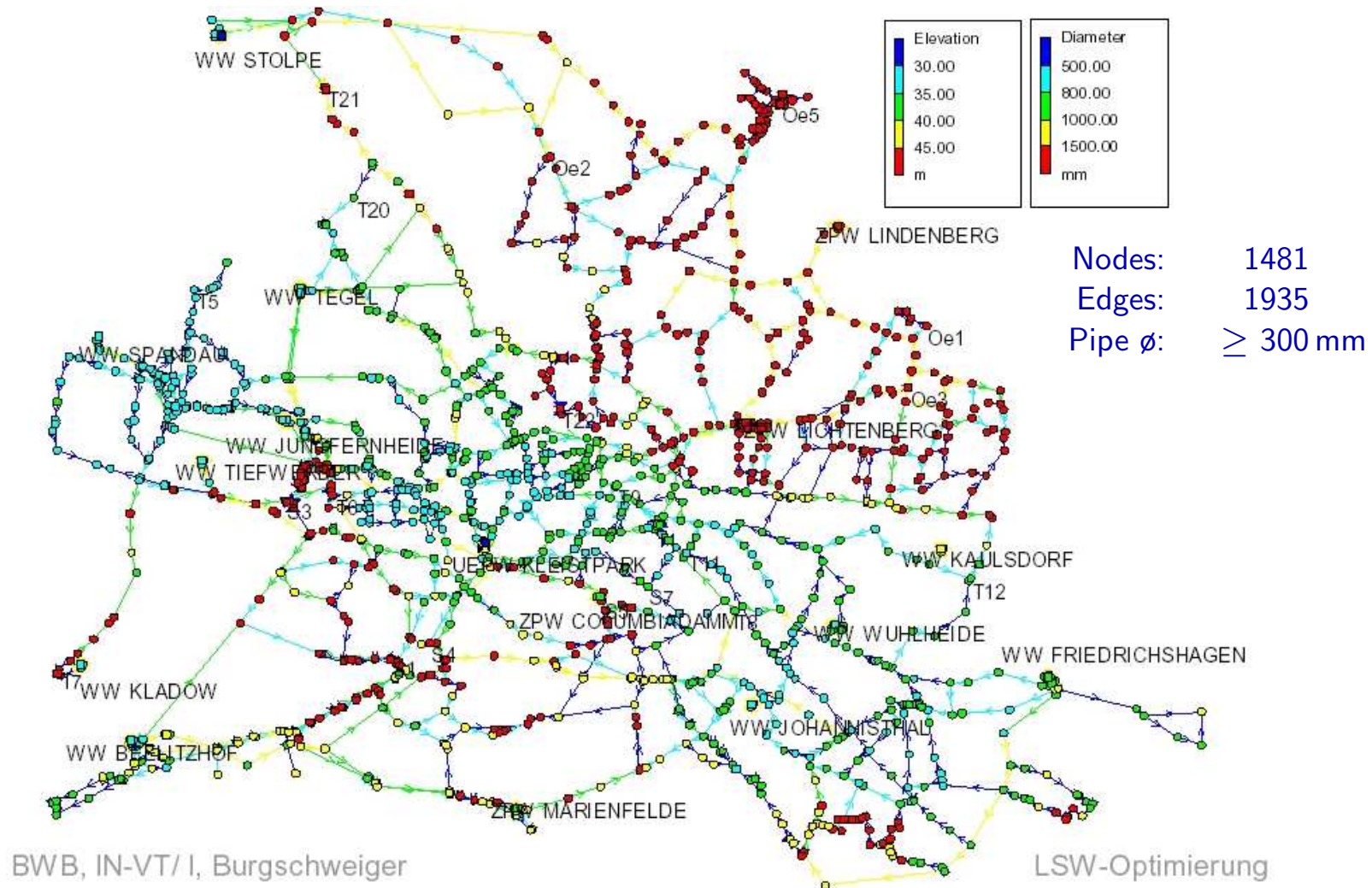
Find: Operating schedule of network satisfying the predicted demand subject to physical, technical, contractual, and other restrictions

Goal: Reduction of daily operating cost



Application

BWB Main Network



Problem Classes

Classical Network Flow

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variable	Equation
Arc	flow	–
Node	–	flow balance
Node: source	–	given inflow
Node: sink	–	given outflow

+ costs, capacities \rightarrow LP (MaxFlow, MinCostFlow)

Problem Classes

Incompressible Network Flow (Water Distribution)

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variable	Equation
Arc	flow	–
Node	–	flow balance
Node: source	–	given inflow
Node: sink	–	given outflow

?

Problem Classes

Incompressible Network Flow (Water Distribution)

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variable	Equation
Arc	flow	–
Node	–	flow balance
Node: source	–	given inflow
Node: sink	–	given outflow

too simple

Additional state variable: pressure

$$\text{head} = \underbrace{\frac{p}{g\rho}}_{\text{hydraulic pressure}} + \text{geodetic height}$$

$\frac{p}{g\rho}$ (mAq)

Problem Classes

Incompressible Network Flow (Water Distribution)

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variable	Equation
Arc: pipe	flow	pressure loss
Node	head	flow balance
Node: source	head	given head
Node: sink	head	given outflow

head = pressure = potential

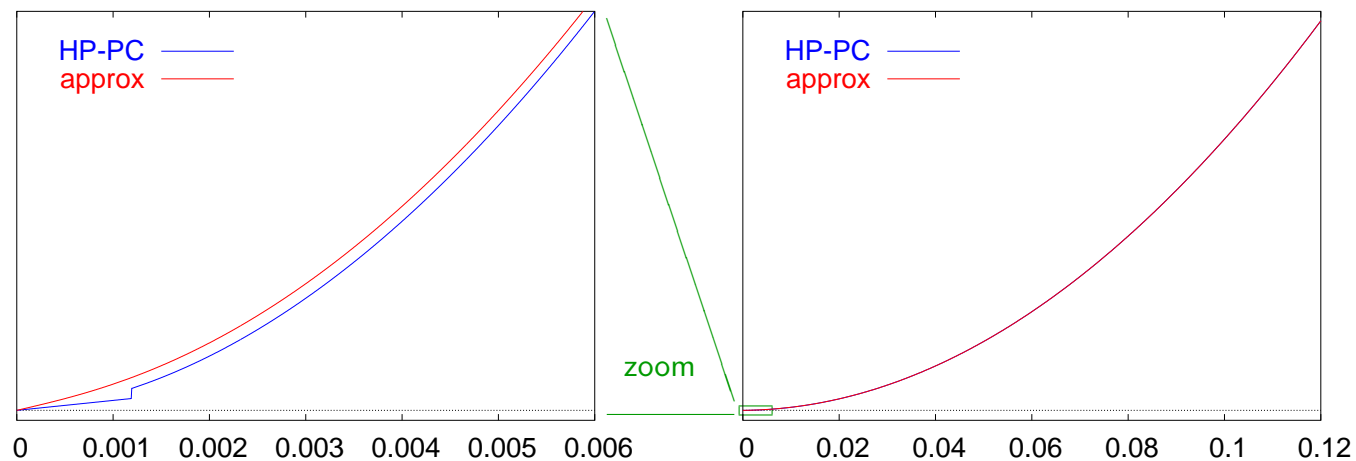
Problem Classes

Incompressible Network Flow (Water Distribution)

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variable	Equation	Type
Arc: pipe	flow Q	pressure loss	nonlinear
Node	head H	flow balance	linear
Node: source	head H	given head	linear
Node: sink	head H	given outflow	linear

head = potential



Problem Classes

Incompressible Network Flow (Water Distribution)

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variables	Equations	Type	Control
Arc	Q, \dots	pressure diff, . . .		
Node	H	flow balance		
Node: source	H	given head	linear	
Node: sink	H	given outflow	linear	
Node: tank	H	flow balance	nonlinear	
Arc: pipe	Q	pressure loss	nonlinear	
Arc: valve	Q	pressure diff	linear	0/1
Arc: ctrl valve	$Q, \Delta H$	pressure decrease	linear	contin.
Arc: pump	$Q, \Delta H, \dots$	pressure increase, . . .	nonlinear	0/1, contin.

Temporal coupling: tanks, min up/down times

Problem Classes

Compressible Network Flow (Gas Distribution)

Digraph $G = (\mathcal{N}, \mathcal{A})$

Element	Variables	Equations	Type	Control
Arc	Q, \dots	pressure diff, ...		
Node	p	flow balance		
Node: source	p	given pressure	linear	
Node: sink	p	given outflow	linear	
Node: tank	p	flow balance	nonlinear	
Arc: pipe	$Q(x), p(x)$	pressure loss, continuity	PDE	
Arc: valve	Q	pressure diff	linear	0/1
Arc: ctrl valve	$Q, \Delta p$	pressure decrease	linear	contin.
Arc: compressor	$Q, \Delta p, \dots$	pressure increase, ...	nonlinear	0/1, contin.

Temporal coupling: tanks, pipes, min up/down times

No potential!



Problem Classes

Hardware



← Drinking water network



Household connection →

Problem Classes

Hardware



← Drinking water network



Household connection →

This is *not* Berlin, it is Tenerife!

Photographs by Mr. Bald (Stadtwerke Plettenberg) <http://www.wasser.de>

Problem Classes

MINLP

Difficulties

- Large: $10^4 - 10^6$ variables, $10^2 - 10^4$ binary
BWB: 1481 nodes, 1935 arcs, ≈ 80 pumps in 17 stations, 24 timesteps
- Substantial nonlinearities
- Disconnected feasible set
pump flow $Q \in \{0\} \cup [Q^-, Q^+]$
pump speed $n \in \{0\} \cup [n^-, n^+]$
- No good lower bounds for B&B
- Expensive NLP relaxations

Reformulation Techniques

Water Networks

Essential for Success

- Approximation of hydraulic pressure loss (asymptotically correct, globally smooth)
- Reduction of network graph (parallel pipes, pipe sequences, small subnetworks)
- Initial estimate by Sequential Linear Programming
- Automatic feasibility analysis via penalty approach
- ▷ Aggregation of pumps (switching); disaggregation in postprocessing
- ▷ Preventing undesirable pump switching (min up/down times)
- ▷ Complementarity constraints for alternative outlets
(Scheel, Scholtes 2000 . . . , Fletcher, Leyffer 2002 . . . , Ragnathan, Biegler 2003)

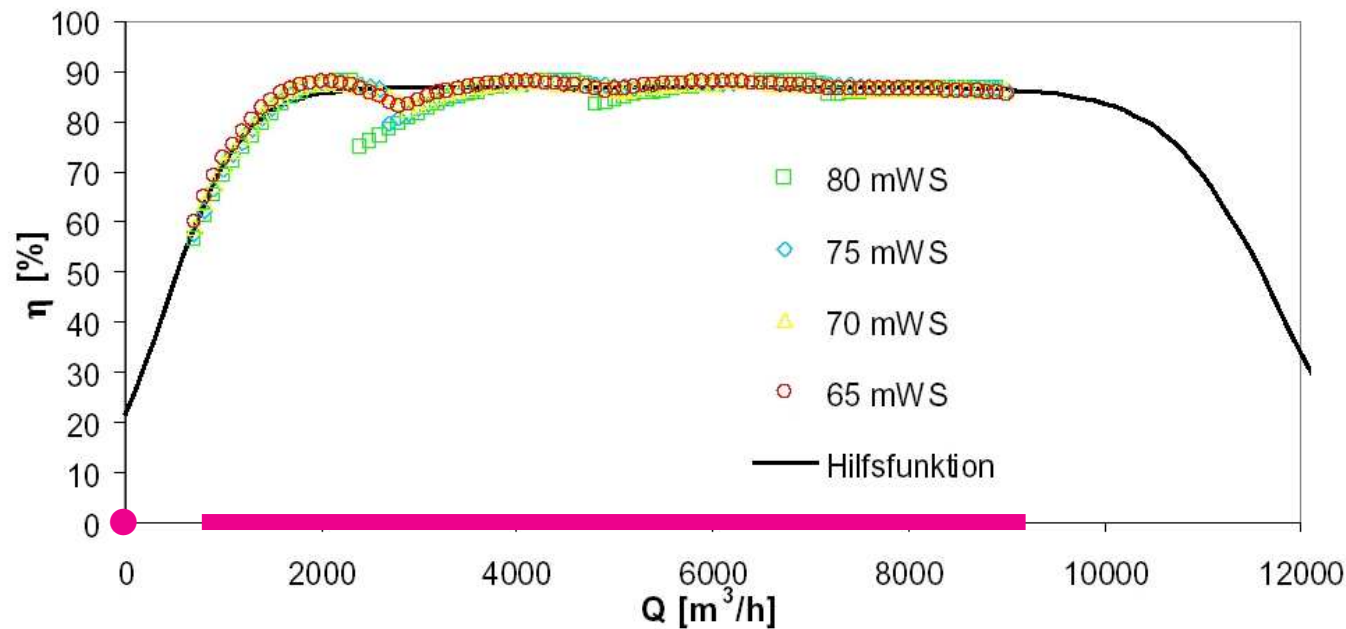
Reformulation Techniques

Pump Aggregation

BWB

- Waterworks have 3 – 6 parallel outlet pumps
- Model as single aggregated unit with $Q = \sum Q_\nu \in \{0\} \cup [\min_\nu Q_\nu^-, \sum_\nu Q_\nu^+]$
- Replace **power model + switching** with **efficiency model**

Combined Efficiency of Pump Collection



Reformulation Techniques

Min Up/Down Times

Typical but Undesired Solution Behavior

- Activation or deactivation of waterworks for just 1 or 2 periods
 - Alternating outflow from two or more waterworks
- Enforce minimum up/down times of 2 or 3 periods



Reformulation Techniques

Min Up/Down Times

Preventing Undesired Pump Switching

- Linear restriction to avoid 0-1-0: lower bound on discrete curvature

$$\begin{aligned}(\Delta t)^2 \ddot{Q}_{t+1} &\approx Q_t - 2Q_{t+1} + Q_{t+2} \\ &\geq -c(Q_t + Q_{t+1} + Q_{t+2}), \quad T = 0, \dots, T - 2\end{aligned}$$

or

$$(c + 1)Q_t + (c - 2)Q_{t+1} + (c + 1)Q_{t+2} \geq 0, \quad T = 0, \dots, T - 2$$

- Parameter c ?



Reformulation Techniques

Min Up/Down Times

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or

$$(c + 1)Q_t + (c - 2)Q_{t+1} + (c + 1)Q_{t+2} \geq 0, \quad T = 0, \dots, T - 2$$

- Parameter $c < 2$ definitely prevents 0-1-0. Interference with other constraints?

$$\begin{aligned}Q_t &\in [Q_t^-, Q_t^+] && t = 1, \dots, T \\ Q_t - Q_{t-1} &\in [\Delta Q_t^-, \Delta Q_t^+] && t = 1, \dots, T\end{aligned}$$

Reformulation Techniques

Min Up/Down Times

Preventing Undesired Pump Switching

- Linear restriction to avoid 0-1-0: lower bound on discrete curvature

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- Parameter $c < 2$ definitely prevents 0-1-0. Interference with other constraints?

$$Q_t \in [Q_t^-, Q_t^+] \quad t = 1, \dots, T$$

$$Q_t - Q_{t-1} \in [\Delta Q_t^-, \Delta Q_t^+] \quad t = 1, \dots, T$$

- No feasible solutions cut off if $c \geq c_{\min}$ where $c_{\min} \in [\frac{1}{2}, 2)$ for reasonable pump design



Reformulation Techniques

Min Up/Down Times

Preventing Undesired Pump Switching

- Linear restriction to avoid 0-1-0 or 0-1-1-0: lower bound on discrete curvature

$$(c_1 + 1)Q_t + (c_1 - 2)Q_{t+1} + (c_1 + 1)Q_{t+2} \geq 0, \quad c_1 \in \left[\frac{1}{2}, 2\right)$$

$$(c_2 + 1)Q_t + (c_2 - 1)Q_{t+1} + (c_2 - 1)Q_{t+2} + (c_2 + 1)Q_{t+3} \geq 0, \quad c_2 \in \left[\frac{1}{3}, 1\right)$$



Reformulation Techniques

Min Up/Down Times

Preventing Undesired Pump Switching

- Linear restriction to avoid 0-1-0 or 0-1-1-0: lower bound on discrete curvature

$$(c_1 + 1)Q_t + (c_1 - 2)Q_{t+1} + (c_1 + 1)Q_{t+2} \geq 0, \quad c_1 \in [\frac{1}{2}, 2)$$

$$(c_2 + 1)Q_t + (c_2 - 1)Q_{t+1} + (c_2 - 1)Q_{t+2} + (c_2 + 1)Q_{t+3} \geq 0, \quad c_2 \in [\frac{1}{3}, 1)$$

- Piecewise linear restriction to avoid 1-0-1:

$$Q_{t+1} \geq c \min(Q_t, Q_{t+2}) - \epsilon, \quad \epsilon \approx \frac{1}{10}Q_-, \quad c \in [0, 1)$$

$$\iff Q_{t+1} - c(Q_t + Q_{t+2} - |Q_t - Q_{t+2}|) + \epsilon \geq 0, \quad c \in [0, \frac{1}{2})$$



Reformulation Techniques

Min Up/Down Times

Preventing Undesired Pump Switching

- Linear restriction to avoid 0-1-0 or 0-1-1-0: lower bound on discrete curvature

$$(c_1 + 1)Q_t + (c_1 - 2)Q_{t+1} + (c_1 + 1)Q_{t+2} \geq 0, \quad c_1 \in [\frac{1}{2}, 2)$$

$$(c_2 + 1)Q_t + (c_2 - 1)Q_{t+1} + (c_2 - 1)Q_{t+2} + (c_2 + 1)Q_{t+3} \geq 0, \quad c_2 \in [\frac{1}{3}, 1)$$

- Piecewise linear restriction to avoid 1-0-1 or 1-0-0-1 (C^0 : very slow!)

$$Q_{t+1} - c(Q_t + Q_{t+2} - |Q_t - Q_{t+2}|) + \epsilon \geq 0, \quad c \in [0, \frac{1}{2}), \quad \epsilon \approx \frac{1}{10}Q_{\min}$$

$$Q_{t+1} - c(Q_t + Q_{t+3} - |Q_t - Q_{t+3}|) + \epsilon \geq 0$$

$$Q_{t+2} - c(Q_t + Q_{t+3} - |Q_t - Q_{t+3}|) + \epsilon \geq 0$$



Reformulation Techniques

Min Up/Down Times

Preventing Undesired Pump Switching

- Linear restriction to avoid 0-1-0 or 0-1-1-0: lower bound on discrete curvature

$$(c_1 + 1)Q_t + (c_1 - 2)Q_{t+1} + (c_1 + 1)Q_{t+2} \geq 0, \quad c_1 \in [\frac{1}{2}, 2)$$

$$(c_2 + 1)Q_t + (c_2 - 1)Q_{t+1} + (c_2 - 1)Q_{t+2} + (c_2 + 1)Q_{t+3} \geq 0, \quad c_2 \in [\frac{1}{3}, 1)$$

- Smoothing: nonlinear restriction to avoid 1-0-1 or 1-0-0-1:

$$Q_{t+1} - c \left(Q_t + Q_{t+2} - \sqrt{(Q_t - Q_{t+2})^2 + \alpha^2} \right) + \epsilon \geq 0, \quad c \in [0, \frac{1}{2})$$

$$Q_{t+1} - c \left(Q_t + Q_{t+3} - \sqrt{(Q_t - Q_{t+3})^2 + \alpha^2} \right) + \epsilon \geq 0, \quad \epsilon \approx \frac{1}{10} Q_{\min}$$

$$Q_{t+2} - c \left(Q_t + Q_{t+3} - \sqrt{(Q_t - Q_{t+3})^2 + \alpha^2} \right) + \epsilon \geq 0$$

→ Smoothed minimum function



Reformulation Techniques

Gas Networks

Compressor Switching

- Formulate MINLP with power constraints $P_t \in [Y_t P_t^-, Y_t P_t^+]$
- Split $Y_t \in \{0, 1\} \rightarrow (Y_t, Z_t) \in \{(0, 1), (1, 0)\}$
- Use NLP reformulations with constraint qualification (Stein, Oldenburg, Marquardt 2003)

$$\begin{array}{ll} Y_t Z_t \leq \mu & \phi_{FB}(Y_t, Z_t) = 0 \\ Y_t + Z_t = 1 & Y_t + Z_t = 1 \end{array}$$

Reformulation Techniques

Gas Networks

Compressor Switching

- Formulate MINLP with power constraints $P_t \in [Y_t P_t^-, Y_t P_t^+]$
- Split $Y_t \in \{0, 1\} \rightarrow (Y_t, Z_t) \in \{(0, 1), (1, 0)\}$
- Use **NLP reformulations** with **constraint qualification** (Stein, Oldenburg, Marquardt 2003)

$$\begin{array}{ll} Y_t Z_t \leq \mu & \phi_{FB}(Y_t, Z_t) = 0 \\ Y_t + Z_t = 1 & Y_t + Z_t = 1 \end{array}$$

- Modification: slowly decrease $\mu = 0.3 \rightarrow 0.01$ with

$$Y_t Z_t \leq \mu, \quad Z_t = \frac{1 - Y_t}{1 + Y_t},$$

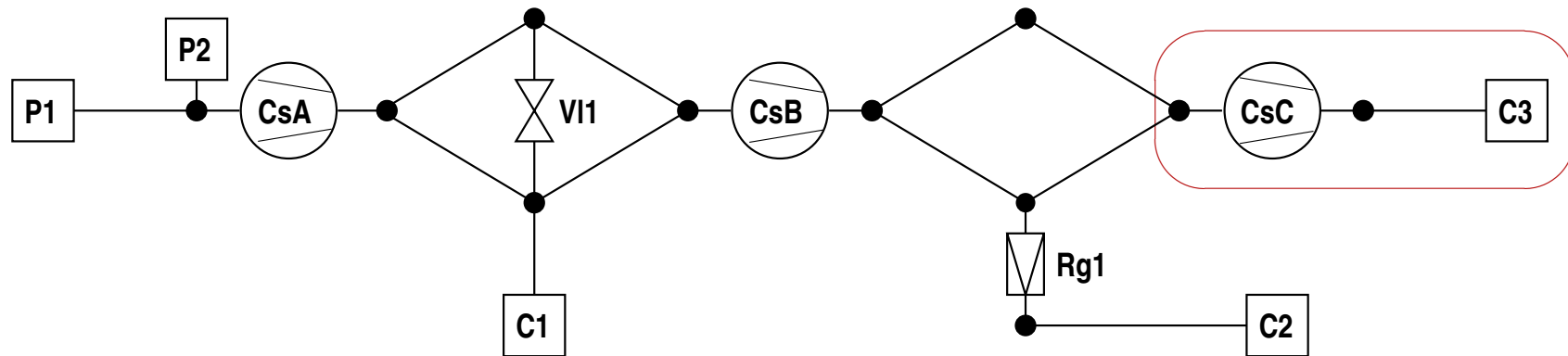
then switch to Fischer-Burmeister formulation

$$\phi_{FB}(Y_t, Z_t) = 0, \quad Y_t + Z_t = 1$$

Reformulation Techniques

Gas Networks

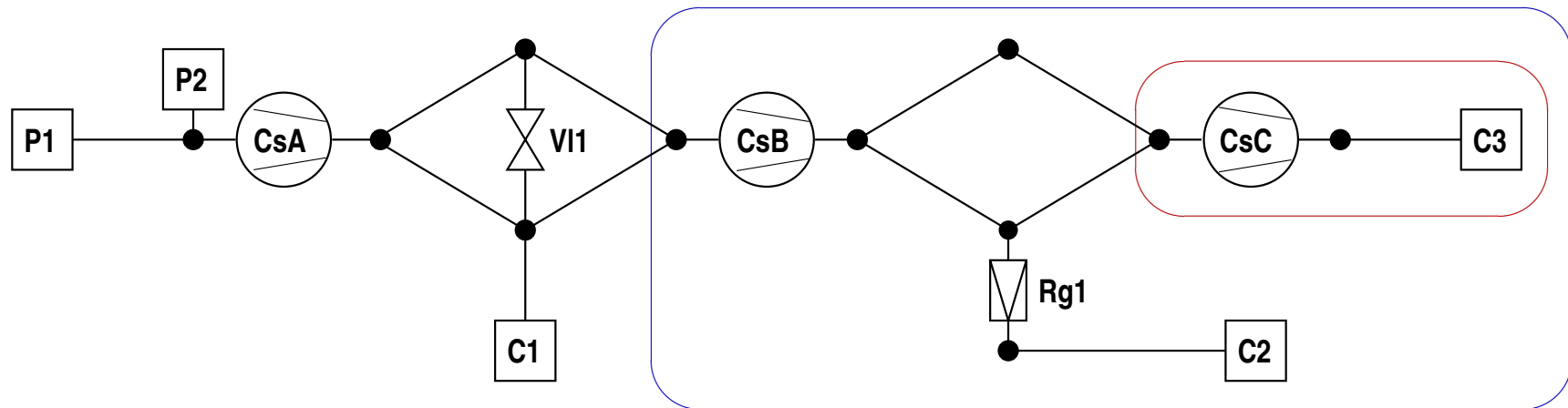
Test Network



Reformulation Techniques

Gas Networks

Test Network

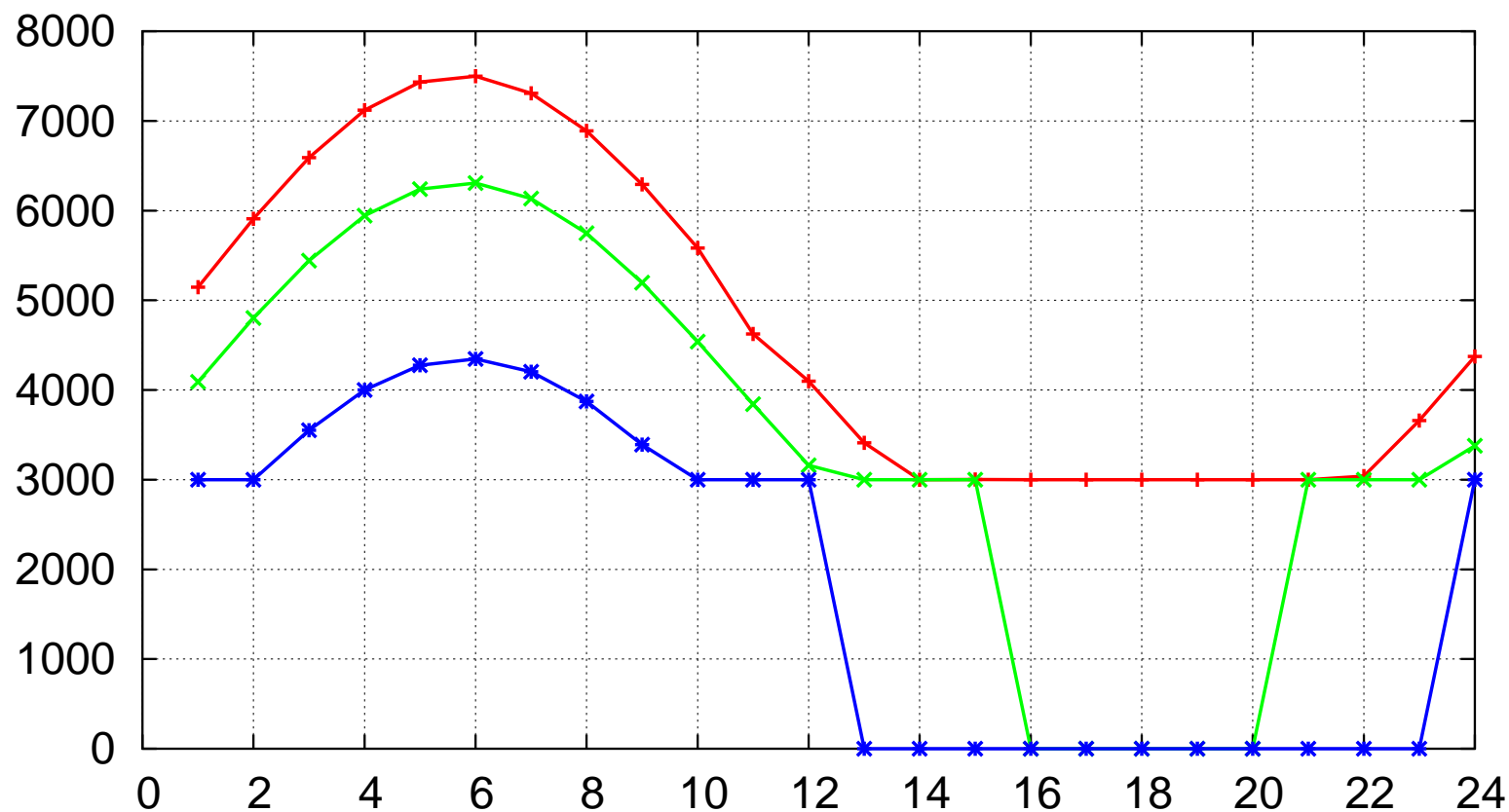


$$\text{cost} = \int_0^T (ap_{in}Q_{in} + c_C P_C + c_B P_B) dt$$

Reformulation Techniques

Gas Networks

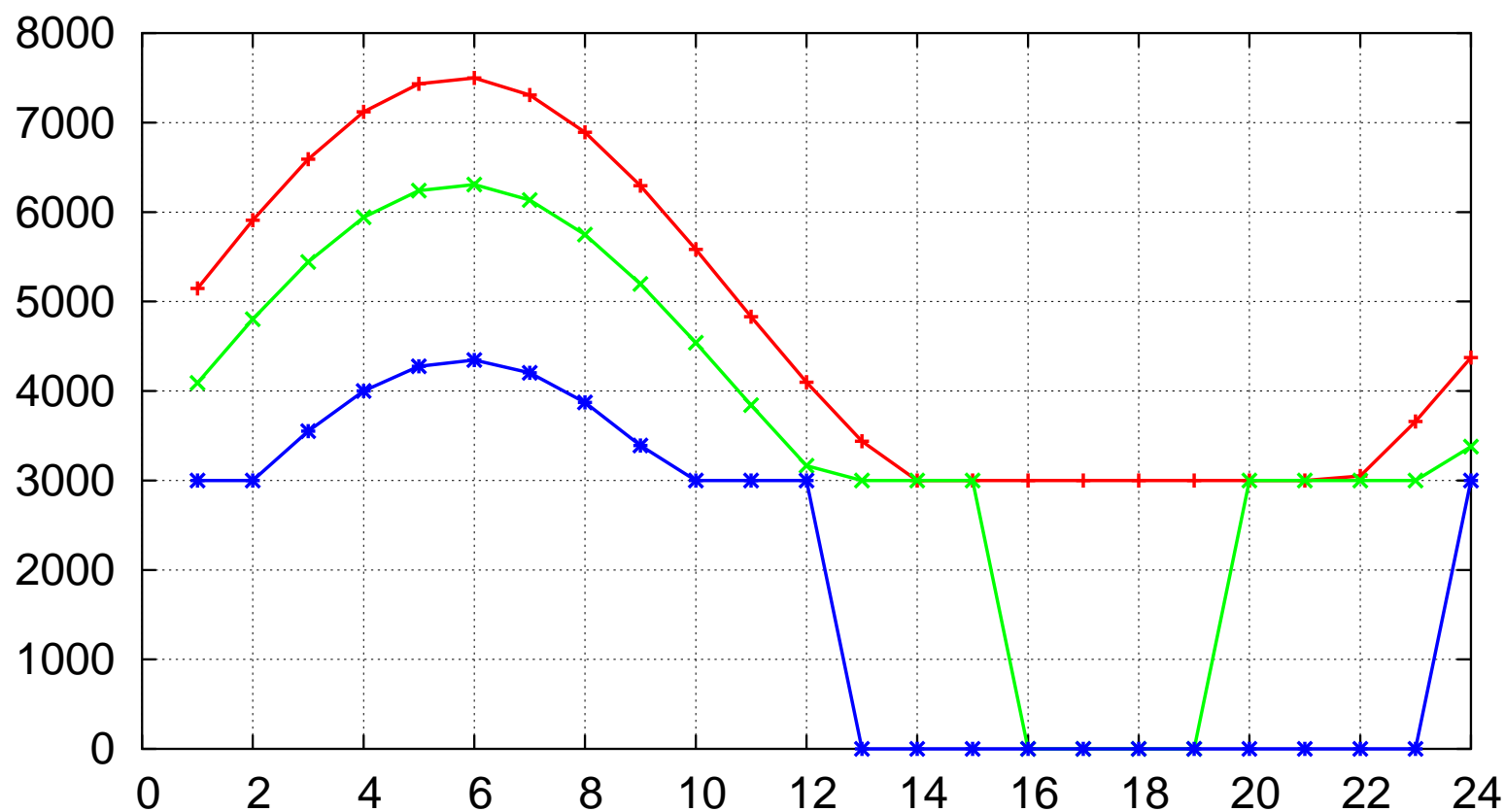
Compressor Power with GAMS DICOPT(MINOS, CPLEX) – red subnetwork



Reformulation Techniques

Gas Networks

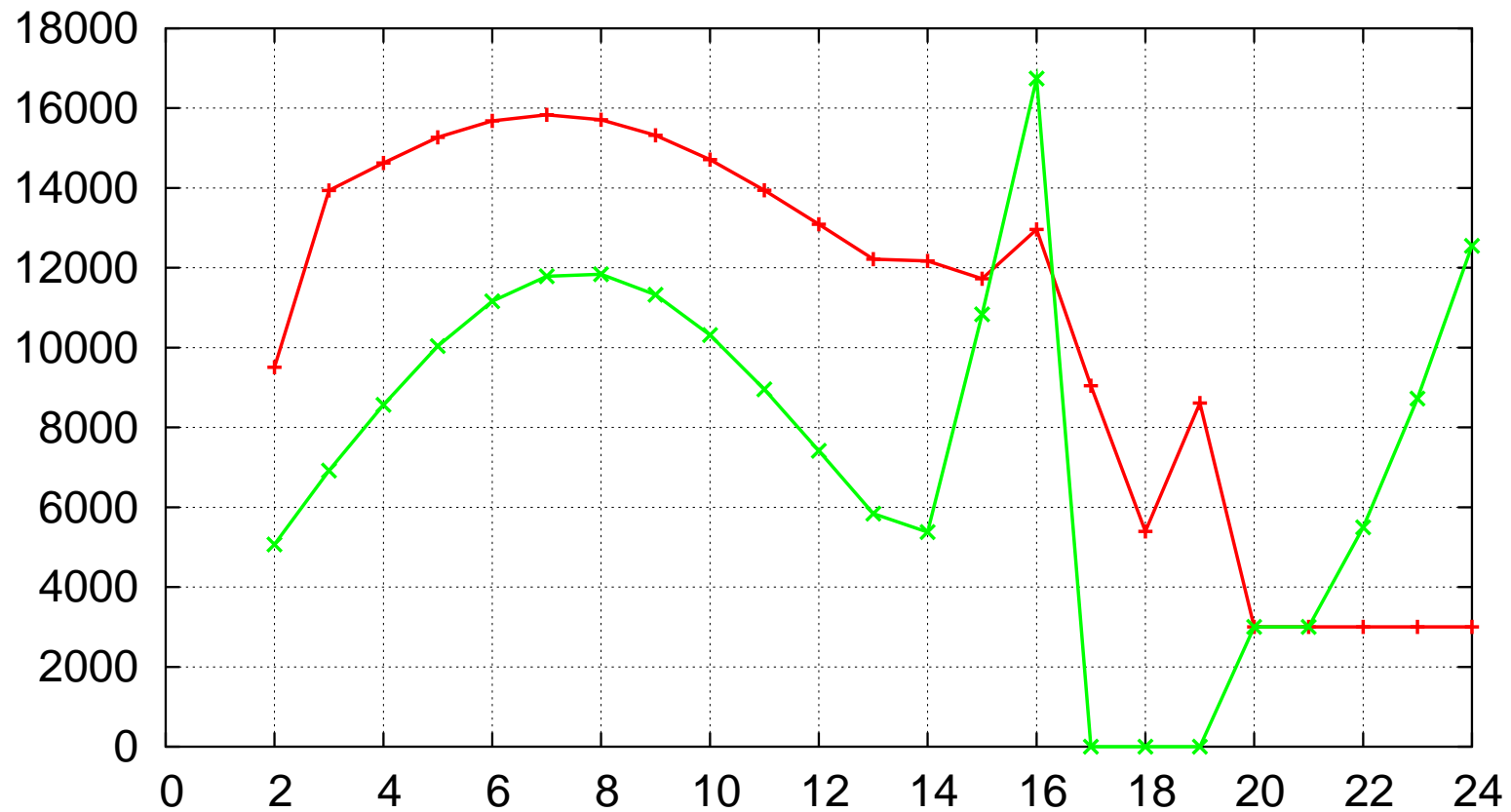
Compressor Power with NLP Reformulation – red subnetwork



Reformulation Techniques

Gas Networks

Compressor Powers with NLP Reformulation – blue subnetwork



Reformulation Techniques

Gas Networks

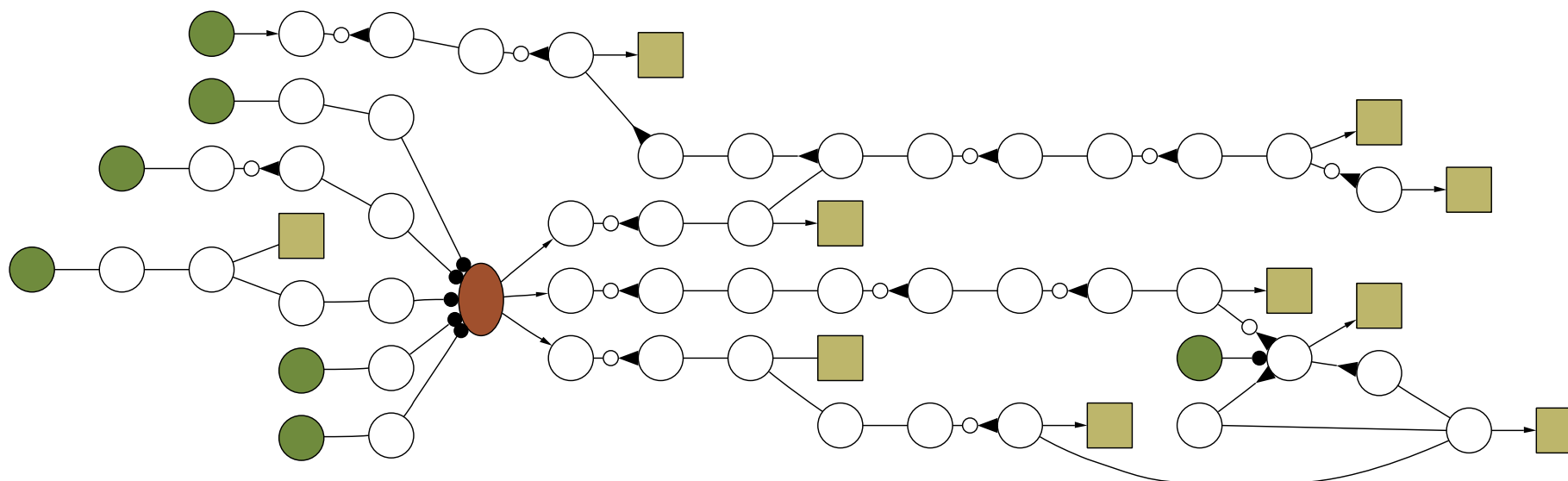
Summary

- **Good results:** almost the same binary decisions as with DICOPT(MINOS, CPLEX)
- **Slow:** μ homotopy needs 30 – 50 NLP
- **No min up/down times**
- **Large networks?**

Reformulation Techniques

Real Gas Networks

Ruhrgas Backbone Network

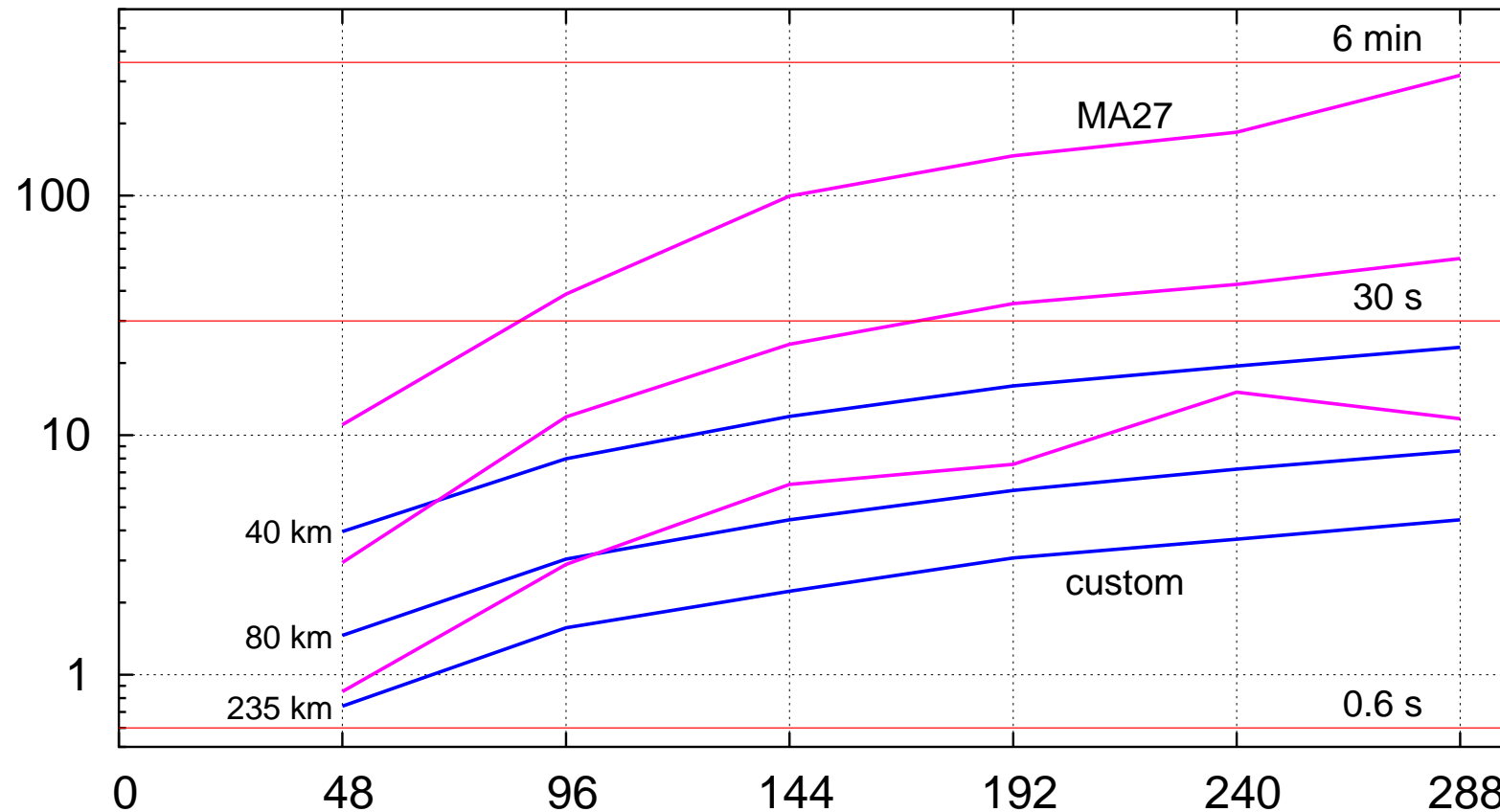


No switching yet – but efficient KKT solver for IPM

Reformulation Techniques

Real Gas Networks

KKT solver: CPU Time vs Discretization



Conclusions

Results

- Treat **combinatorial aspects** by NLP techniques
- Gas networks:
 - **Compressor switching** by aggregated efficiency model + reformulation
 - **No min up/down times**
 - **Very small networks**
- Water networks:
 - **Pump switching** by aggregated efficiency model
 - **Minimum up/down times** by linear/smoothed NCP constraints
 - **Alternative outlets** by complementarity constraints
 - Obtain **practically satisfactory solutions**
 - CPU time **15 – 25 minutes** for BWB network
 - Optimization module **in operation since 06/2004**

Challenges

- Custom NLP solver: ✓
- Exact MINLP algorithm?