

# Term End Exam Scheduling at USMA, West Point

September 2001

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**GAMS Development**  
**Corp.**

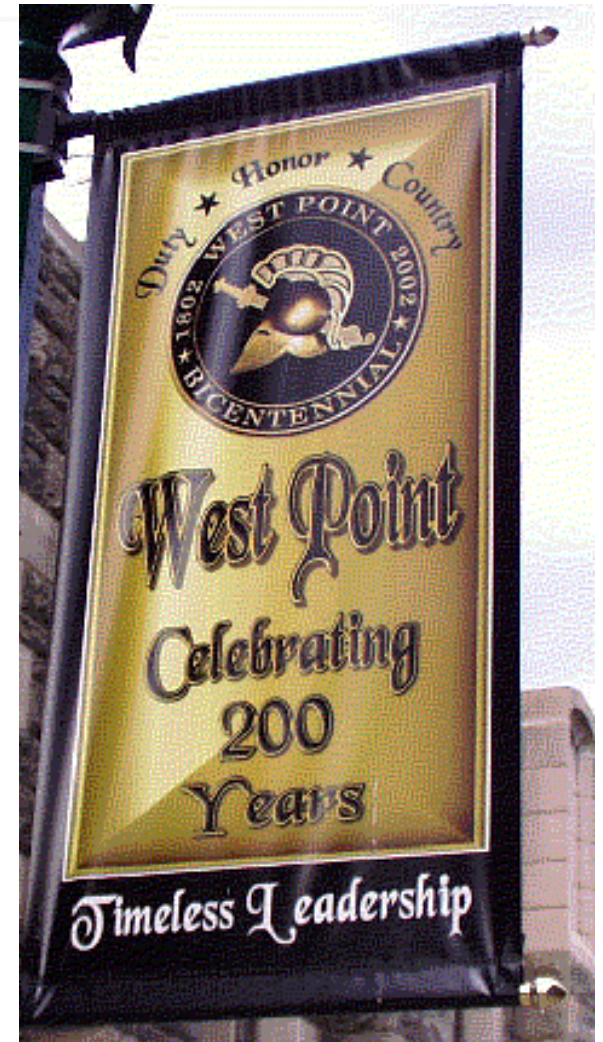
**Fred O'Brien**  
**United States**  
**Military**  
**Academy.**

**Siqun Wang**  
**Wharton School**  
**Univ. of Pennsylvania**

# USMA Mission

To educate, train, and inspire the  
Corps cadets so that each  
graduate is a commissioned leader  
of character committed to the  
values of the United States  
professionals throughout a  
career as an officer in the United  
States Army; and a lifetime of  
selfless service to the nation.

**EXAMS**



# Academic Scheduling

- Term end exam (TEE) scheduling
  - Scheduling preparation
  - Find *good* schedules for exam courses **and** cadets.
  - Post scheduling
- Course scheduling
  - For a given set of *course offerings* find *good* schedules for all cadets.

**Course Tee Offerings**

- All
- Offering Tee
- No Tee

Courses with Firsties:

Course Search:

**2001 - 2  
 COURSE OFFERINGS  
 225 COURSES**

COURSE	#SECT	ENRLD	CRSE DIRECTOR	TEE YES	TEE NO
<a href="#">CE371</a>	3	<a href="#">48</a>	DR EVANS	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CE380</a>	3	<a href="#">48</a>	MAJ MCDONALD	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CE403</a>	3	<a href="#">52</a>	CPT NAKANO	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CE404</a>	4	<a href="#">79</a>	MAJ SCHIRNER	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CE490</a>	3	<a href="#">42</a>	CPT LACHANCE	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH101</a>	2	<a href="#">21</a>	COL BLACKMAN	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH102</a>	49	<a href="#">891</a>	MAJ CHAPMAN	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH152</a>	7	<a href="#">138</a>	MAJ MALONE	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH381</a>	1	<a href="#">11</a>	MAJ CHOUNG	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH384</a>	4	<a href="#">44</a>	COL DOOLEY	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH401</a>	1	<a href="#">11</a>	DR BIAGLOW	<input checked="" type="radio"/>	<input type="radio"/>
<a href="#">CH457</a>	4	<a href="#">39</a>	DR LABARE	<input checked="" type="radio"/>	<input type="radio"/>

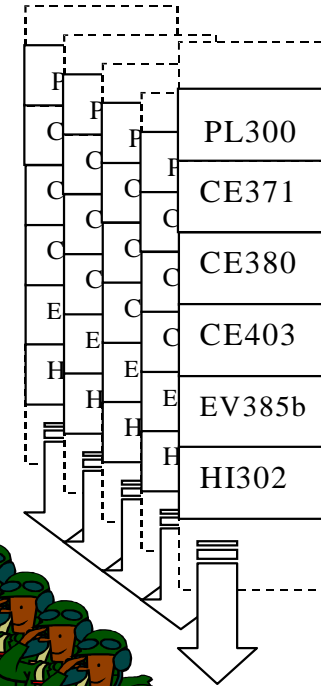
[Set Acad Year](#) [Set Periods](#) [Main](#)

**2001 - 2 TEE PERIODS**

Year	Term	Date	Period	Start Time	End Time
2001	2	05/21/01	1	0735	1105
2001	2	05/21/01	3	0735	1105
2001	2	05/21/01	2	1500	1830
2001	2	05/22/01	5	0735	1105
2001	2	05/22/01	4	1500	1830
2001	2	05/23/01	6	1500	1830
2001	2	05/24/01	7	0735	1105
2001	2	05/24/01	8	1500	1830
2001	2	05/25/01	9	0735	1105
2001	2	05/25/01	10	1500	1830
2001	2	05/26/01	11	0735	1105
2001	2	05/26/01	12	1500	1830

# Term End Exam Scheduling

	1	2		6
morning period	CE371	CH101		EV203
	CH384	CS408		PH203
	CS383	EE301		PL300
	HI366	EN302		LR204
afternoon period	CE404	LF382		CE403
	LG484	SE388		CS380
	LS362	SS388		SS201
	MS350	...		...



Term End Exam Courses ( $\approx 250$  courses)

$\approx 4000$  cadets

$\approx 20000$  exams

# Overcoming Conflicts

- Schedule with conflicts

1	2		6
CE371 CH384 CS383 HI366	CH101 CS408 EE301 EN302		EV203 PH203 PL300 LR204
CE404 LG484 LS362 MS350	LF382 SE388 SS388 CE403		CE403 CS380 SS201 ...



Cadet's 8TAP:

PL300
CE372
CE403
CS380
EV180
HI302

- Makeup/ahead for an exam course:
  - An additional exam offering for a small group of cadets who can not go to the primary exam offering
- Resolve conflicts by adding makeup/ahead

# TEE Scheduling

- Given exam courses
  - MA481 CE371 CH100 ...
- Given exam periods
  - p1, p2, p3, ... p12
- Given cadet's exam course '*requests*'
  - 043671571,CE403
  - 043671571,CE380
- Find an assignment of exam course sessions (primaries, makeups) to periods and cadet's requests to exam courses sessions.
  - CE403,prim,p12 CE403,mkup,p4 CE380,prim,p4 ...
  - 043671571,CE403,p4 043671571,CE380,p4 ...
- Objective: Minimize the total number of makeups



# TEE Constraints

- No conflicts (no cadet in more than one exam sessions per period) (Conflict)
- All exam courses and all exam requests must be assigned (Assign)
- Parts of the exam session schedule (i.e. fixed periods for some primaries) may be given (Fixed)
- Primary exam session enrollment minimum (e.g. 75%) (PrimEnroll)
- Certain periods are prohibited for certain exam courses. (Prohibited)

# More TEE Constraints

- Inclusive/Exclusive groups
  - Some primaries must be scheduled together (Inclusive)
  - Some exam courses must be scheduled apart (Exclusive)
- Courses with no makeups allowed (No makeups)
- Plebe/Firsties constraints
  - One exam a day for Plebes (Exams per day)
  - Exam period is over by period X for Firsties (Finished)
- Consecutive (back to back) exam limit (Consecutive)
- Produce schedules in a timely fashion

# An Optimization Model

## ■ Variables

- $x(c, r, p)$  cadet request to course/period 250.000
- $y(r, s, p)$  course session to period |S| \* 3.000
- $z(r, p)$  primary indicator 3.000

## ■ Constraints

- Conflict 50.000
- Assign 20.000
- PrimEnroll  $\sum_c x(c, r, p) \geq 0.75 \cdot enroll(r) \cdot z(r, p), \sum_p z(r, p) = 1$  3.250
- Consecutive 36.000
- Exams per day 6.000
- Inclusive |I| \* 12
- Exclusive |S| |I| \* 12
- Fixed, Prohibit, No makeup, Finished by variable fixing
- Coupling of x and y  $\sum_c x(c, r, p) \leq enroll(r) \cdot y(r, s, p)$  |S| \* 3.000

# Analysis of this Model

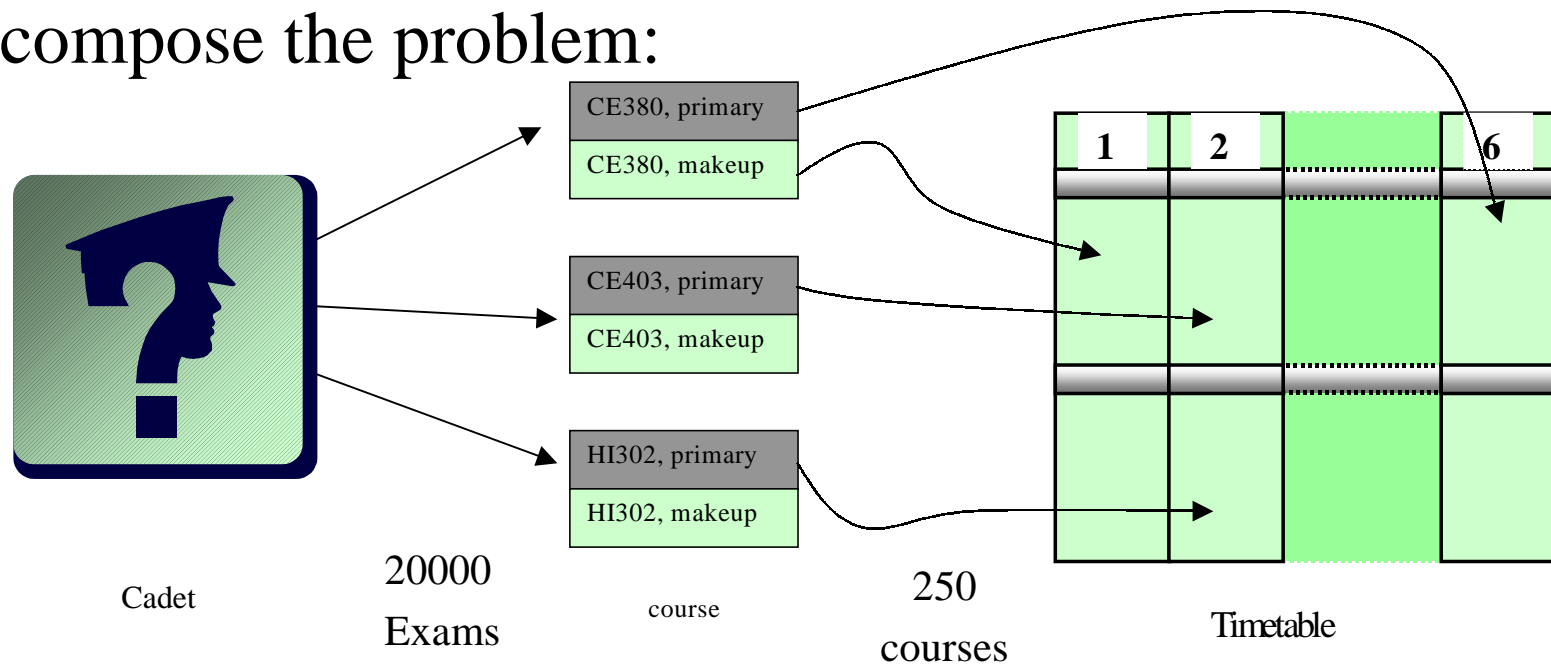
- `Consecutive Constraint` is a headache
  - Many constraints, lots of non zeros
  - ‘Unimportant’ constraint
  - ➡ drop `Consecutive constraint` from model
- Problem is large
- Problem is (integer) infeasible
- Model with penalized violations expensive
  - CPLEX 7.1 doesn’t find any integer solution for a feasible model within 24h on a 1GHz PC
  - CPLEX 7.1 doesn’t improve a given bad integer solution (using **`mipstart`**) within 24h on a 1GHz PC

# Solution Approach

- Heuristic based on a collection of medium sized optimization models produces conflict free schedules and automatically relaxes constraints.
- Improvement module starts with a good/mediocre solution and a set of relaxed constraints and tries to
  - Improve number of makeups
  - Reinforce relaxed constraints

# Solution Improvement

- Given a solution
  - Assignment of exam courses (primary and makeup) to periods
  - Assignment of cadets to periods
  - Set of relaxed constraints
- Decompose the problem:



# Improvement Model 1

- Fixed assignment of cadet request to exam course session
  - Variable transformation  $x(c, r, p) \sim x(c, r, s)$
- Variables
  - $w(r, p)$  open period for course session 3.000
  - $y(r, s, p)$  course session to period  $|S'| * 3.000$
- Constraints
  - Conflict  $\sum_{r,s} y(r, s, p) \leq 1$  50.000
  - Exams per day 6.000
  - Inclusive/Exclusive  $(1 + |S'|) |I| * 12$
  - Fixed, Prohibit, No makeup, Finished by variable fixing
  - Coupling of  $w$  and  $y$  3.000
- Objective  $y(r, s, p) \leq w(r, p)$ 
  - Min  $\sum((r, p), w(r, p))$  (folds sessions of the same course)
  - Min violations of relaxed constraints

# Improvement Model 2

- Fixed assignment of exam sessions to periods
- Variables
  - $x(c, r, s)$  cadet request to course session 250.000
  - $y(r, s, p)$  course session open  $|S'| + 250$
- Constraints
  - Conflict 20.000 + a(S')
  - Exams per day 6.000
  - Finished by variable fixing
  - Coupling of  $x$  and  $y$   $|S'| + 250$
- Objective
  - Min  $\sum((r, s, p), y(r, s, p)) + \text{enrollment in makeup}$   
(closes some sessions and reduces enrollment in makeup)



# Improvement Algorithm

- The model 1 and 2 are iteratively repeated
- Model 1 requires a good/mediocre solution
  - Size and running time depend on number of makeups in the initial solution (need heuristic)
- The different cadet/session assignment makes new course/period assignment likely and vice versa.
  - The constraint reinforcing objective shocks the system if it is stuck
- Terminates when out of time (user input) or shocking doesn't help

SEARCH FOR CADET:

Enter Cadet's Last Name:  OK

Enter Cadet SSN:  OK

SEARCH BY COURSE:

Course Search:  OK

Tee Course Matrix:

### 2001 - 2 Term End Course Matrix

	1	2	3	4	5	6	7	8	9
	05/21/2001	05/21/2001	05/21/2001	05/22/2001	05/22/2001	05/23/2001	05/24/2001	05/24/2001	05/25/2001
	<a href="#">CH474</a> 13	<a href="#">CE404</a> 79	<a href="#">CH101</a> 21	<a href="#">CH472B</a> 31	<a href="#">CE371</a> 47	<a href="#">CH384</a> 1	<a href="#">CE490</a> 42	<a href="#">CH490</a> 17	<a href="#">CE380</a>
	<a href="#">CS360</a> 1	<a href="#">CS478</a> 19	<a href="#">CH102</a> 890	<a href="#">CH486</a> 41	<a href="#">CH381</a> 11	<a href="#">EE301</a> 31	<a href="#">CH478</a> 42	<a href="#">EE381</a> 1	<a href="#">CH384</a>
	<a href="#">CS380</a> 1	<a href="#">EE478</a> 13	<a href="#">CH152</a> 138	<a href="#">CS380</a> 20	<a href="#">CH478</a> 1	<a href="#">EM362</a> 2	<a href="#">CS385</a> 22	<a href="#">EM486</a> 4	<a href="#">CS383</a>
	<a href="#">CS383</a> 18	<a href="#">EP362</a> 7	<a href="#">CH401</a> 11	<a href="#">CS482</a> 13	<a href="#">CH486</a> 10	<a href="#">HI301</a> 8	<a href="#">CS476</a> 15	<a href="#">EV365</a> 21	<a href="#">CS408</a>
	<a href="#">EM301</a> 94	<a href="#">EV203</a> 27	<a href="#">CS408</a> 11	<a href="#">EE301</a> 88	<a href="#">CS105</a> 420	<a href="#">HI302</a> 834	<a href="#">EE365</a> 7	<a href="#">EV401</a> 1	<a href="#">CS408</a>
	<a href="#">EM301A</a> 88	<a href="#">EV389H</a> 19	<a href="#">CS484</a> 2	<a href="#">EE475</a> 6	<a href="#">CS155</a> 104	<a href="#">HI302X</a> 39	<a href="#">EE381</a> 51	<a href="#">EV471</a> 9	<a href="#">EE363</a>
	<a href="#">EM362</a> 125	<a href="#">HI383</a> 11	<a href="#">EM478</a> 24	<a href="#">EE484</a> 36	<a href="#">CS360</a> 10	<a href="#">HI352</a> 125	<a href="#">EE471</a> 6	<a href="#">HI382</a> 11	<a href="#">EE363</a>
	<a href="#">EM362A</a> 10	<a href="#">LC382</a> 1	<a href="#">EN302</a> 524	<a href="#">EM380</a> 42	<a href="#">CS360A</a> 68	<a href="#">HI385</a> 14	<a href="#">EM364</a> 119	<a href="#">LA362</a> 14	<a href="#">EE365</a>
	<a href="#">EN302</a> 23	<a href="#">LF492</a> 5	<a href="#">EV389H</a> 1	<a href="#">EV203</a> 413	<a href="#">CS384</a> 19	<a href="#">LA472</a> 1	<a href="#">EM364A</a> 34	<a href="#">LF484</a> 13	<a href="#">EE365</a>
	<a href="#">EV374</a> 1	<a href="#">LR482</a> 6	<a href="#">EV482</a> 43	<a href="#">EV386</a> 18	<a href="#">CS482</a> 2	<a href="#">LC362</a> 1	<a href="#">EM486</a> 33	<a href="#">LG492</a> 17	<a href="#">EE483</a>
	<a href="#">EV385B</a> 5	<a href="#">LS362</a> 6	<a href="#">HI301</a> 1	<a href="#">EV391B</a> 20	<a href="#">EM301</a> 37	<a href="#">MA205</a> 70	<a href="#">EN101</a> 36	<a href="#">LR382</a> 4	<a href="#">EM362</a>
	<a href="#">HI103</a> 3	<a href="#">MA484</a> 2	<a href="#">HI372</a> 4	<a href="#">HI366</a> 33	<a href="#">EM301A</a> 8	<a href="#">MA206</a> 719	<a href="#">EN102</a> 999	<a href="#">LS476</a> 36	<a href="#">EP372</a>
	<a href="#">HI104</a> 281	<a href="#">MA485</a> 8	<a href="#">HI389</a> 43	<a href="#">HI384</a> 16	<a href="#">EM302</a> 170	<a href="#">MA364</a> 1	<a href="#">EP368</a> 15	<a href="#">MA206</a> 13	<a href="#">EV377</a>
	<a href="#">HI107</a> 12	<a href="#">ME480</a> 53	<a href="#">LA204</a> 74	<a href="#">HI390</a> 28	<a href="#">EV365</a> 85	<a href="#">PL375</a> 4	<a href="#">EV385B</a> 106	<a href="#">MA364</a> 122	<a href="#">EV388</a>
	<a href="#">HI108</a> 547	<a href="#">MS350</a> 8	<a href="#">LA484</a> 4	<a href="#">LC362</a> 8	<a href="#">EV388B</a> 27		<a href="#">EV386</a> 2	<a href="#">MA391</a> 40	<a href="#">EV390</a>
	<a href="#">HI154</a> 65	<a href="#">MS489</a> 1	<a href="#">LC204</a> 71	<a href="#">LF484</a> 1	<a href="#">HI371D</a> 36		<a href="#">EV401</a> 11	<a href="#">PL381</a> 63	<a href="#">EV487</a>
	<a href="#">HI158</a> 123	<a href="#">PH374</a> 8	<a href="#">LF204</a> 64	<a href="#">LG362</a> 13	<a href="#">HI377</a> 41		<a href="#">EV482</a> 5	<a href="#">PY201</a> 51	<a href="#">HI372</a>
	<a href="#">HI366</a> 3	<a href="#">PH472</a> 5	<a href="#">LG204</a> 132	<a href="#">LN380</a> 33	<a href="#">HI382</a> 1		<a href="#">HI302</a> 1	<a href="#">SS307</a> 484	<a href="#">HI375</a>
	<a href="#">HI367</a> 21	<a href="#">PL300</a> 476	<a href="#">LP204</a> 81	<a href="#">LR492</a> 6	<a href="#">HI384</a> 3		<a href="#">HI389</a> 2	<a href="#">SS357</a> 44	<a href="#">HI398</a>
	<a href="#">HI383</a> 2	<a href="#">PL385</a> 86	<a href="#">LR204</a> 71	<a href="#">LS492</a> 1	<a href="#">HI398</a> 22		<a href="#">HI498</a> 1	<a href="#">SS376</a> 25	<a href="#">LF362</a>
	<a href="#">LF382</a> 2	<a href="#">SE385</a> 99	<a href="#">LS204</a> 251	<a href="#">MA476</a> 23	<a href="#">LA472</a> 5		<a href="#">LA382</a> 6	<a href="#">SS478</a> 19	<a href="#">LG362</a>
	<a href="#">LP204</a> 1	<a href="#">SE388</a> 16	<a href="#">SS477</a> 36	<a href="#">MA492C</a> 3	<a href="#">LC472</a> 6		<a href="#">LF362</a> 3		<a href="#">LN482</a>

SEARCH FOR CADET:

Enter Cadet's Last Name:

Enter Cadet SSN:

SEARCH BY COURSE:

Course Search:

Tee Course Matrix

PRIVACY ACT DATA - FOR OFFICIAL USE ONLY

Cadet Admin Data

Name SMITH, BENJAMIN DARDEN Pn Id 068722539 Status C Class 2003 Co B3

Email [x35716@usma.edu](mailto:x35716@usma.edu) Fos [MF](#) 2nd FOS Eng Seq EV

CODE DESCRIPTION ADVISOR  
MF Economics MAJ CPT LOVER

**PY201 MAKEUPS**

PY201 7

**2001 - 2 Tee Schedule By Date**

	05/21/2001	05/22/2001	05/23/2001	05/24/2001	05/25/2001	05/26/2001
1	<a href="#">PH204</a>	3 <a href="#">LA204</a>	5	7 <a href="#">PY201</a>	9	11
2	<a href="#">SS252</a>	4 <a href="#">SS388</a>	6	8	10	12

# Feasibility Study

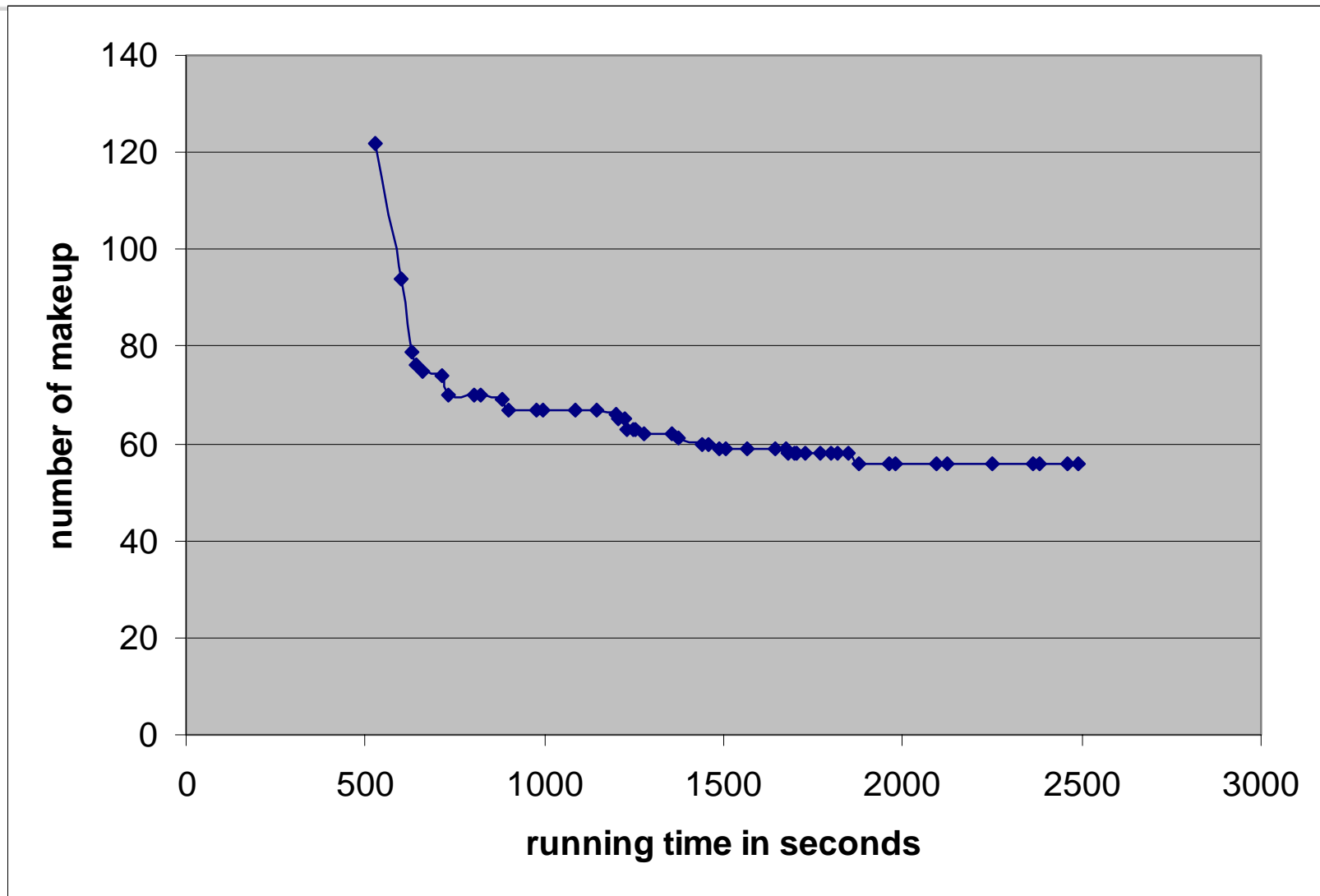
- TEE last application of legacy system
  - Mainframe, Cobol, ~1980
  - Maintenance + on-site personnel: \$500,000/year
- By March 2001: decision for renewal
- TEE Schedule for AY2001/2 (End of May 2001)
  - Chuck + Legacy system
    - Partial schedule, approx. 90 makeups (4 Weeks)
  - Chuck + GAMS TEE scheduler
    - Complete schedule, no conflicts, 60 makeups (10 minutes)
    - The improver module produced schedule with 40 makeups

# More Computational Results

- Three data sets 01/2,02/1(early),02/1
- Constraint violations ‘OK’

Year	Courses	Periods	Requests	Makeups
01/2	226	12	18937	38
02/1 early	213	12	18512	49
02/1	252	11	21175	61

# Improvement over Time



A black and white photograph of a military band marching in a parade, viewed through a blue-tinted lens. The band members are wearing dark uniforms and caps, and are carrying large brass instruments. The text "The End" is overlaid in the center in a yellow, serif font.

The End